DOCUMENT RESUME

ED 207 795 SE 033 656

AUTHOR	Dixon, Peggy; And Others
TITLE	Physics of Mechanical, Gaseous, and Fluid Systems. A Study Guide of the Science and Engineering Technician Curriculum.
INSTITUTION	Saint Louis Community Coll. at Florissant Valley, Mo.
SPONS AGENCY	National Science Foundation, Washington, D.C.
PUB DATE	76
GRANT	NSF-GZ-3378; NSF-HES74-22284-A01; NSF-SED77-17935
NOTE	107p.; For related documents, see SE 033 647-657. Not available in paper copy due to copyright restrictions. Contains occasional light and broken type.
AVAILABLE FROM	National Science Teachers Association, 1742 Connecticut Ave., N.W., Washington, DC 20009 (write for correct price).
EDRS PRICE	MF01 Plus Postage. PC Not Available from EDRS.
DESCRIPTORS	Acoustics; Associate Degrees; *College Science; *Engineering Education; Heat; *Instructional Materials; Interdisciplinary Approach; Measurement; Mechanics (Physics); Motion; *Physics; *Postsecondary Education; Science Course Improvement Projects; *Technical Education; Temperature; Time
IDENTIFIERS	*Science and Engineering Technician Curriculum

ABSTRACT

This study guide is part of a program of studies entitled Science and Engineering Technician (SET) Curriculum. The SET Curriculum integrates elements from the disciplines of chemistry, physics, mathematics, mechanical technology, and electronic technology. The objective of this curriculum development project is to train technicians in the use of electronic instruments and their applications. This guide provides part of the content of the physics component of the curriculum. The document introduces basic concepts such as length (distance), time, mass, weight, measurement, and basic electrical concepts. Other topics include the following: (1) translational motion; (2) rational motion; (3) temperature and heat; (4) properties of gases and liquids; and (5) sound and wave motion. (Author/SK)

SEDR DATA FORMAT	1. Award Number SED77-17935	2. Award Date September 28, 1977	3. Termination Date July 31, 1979

6. Title
Physics of Mechanical Gaseous and Fluid Systems:
A Study Guide of the Science and Engineering Technician Curriculum

4. Amount of Award
$70,400 (Cum. Amt.)

5. Type
Final Technical Report

7. Performing Organization
SET Project
St. Louis Community College at Florissant Valley
St. Louis, MO 63135

8. Pagination
105 pages

9. Accession Number
00318

10. Performing Organization Report Number

11. Principal Investigator; Field or Specialty
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13. SEDR Program
DISE

14. SEDR Subprogram

15. Abstract

This study guide is part of a program of studies entitled the Science and Engineering Technician Curriculum (SET). The SET curriculum integrates elements from the disciplines of chemistry, physics, mathematics, mechanical technology, and electronic technology. The objective of this curriculum development project is to train technicians in the use of electronic instruments and their applications.

This guide provides part of the content of the physics component of the curriculum. The document introduces basic concepts such as length (distance), time, mass, weight, measurement, and basic electrical concepts. Other topics include the following: translational motion; rational motion; temperature and heat; properties of gases and liquids; and sound and wave motion.

16. Descriptors
Study Guides
Instructional Materials
Associate Degrees
Science Curriculum
Engineering Technicians
Physics
Physics Curriculum
Physics Instruction

17. Identifiers
Science and Engineering Technician Curriculum (SET)

18. Field
Physics, General

19. Target Audience
Two-Year College Students

20. Availability
[illegible] ed number of copies available from St. [illegible] Community College at Florissant Valley [illegible] Louis, MO 63135

21. Supplemental Notes
One copy

PHYSICS
OF
MECHANICAL, GASEOUS, AND FLUID SYSTEMS

A STUDY GUIDE
OF
THE SCIENCE AND ENGINEERING TECHNICIAN CURRICULUM

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The materials contained herein were developed under Grant Numbers HES74-22204 A01, GZ-3378, and SED77-17935.

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CHAPTER I

BASIC CONCEPTS

SECTION 1 - LENGTH AND TIME

Length or distance is measured in terms of an arbitrarily defined SI* unit called the meter (m). One meter was originally defined to be 1/10,000,000 of the distance between the North Pole and the Equator, measured along a parallel of longitude through Paris, France. More recently, the meter has been defined in terms of the basic properties of the wavelength of light.

1 meter = 100 centimeters (cm)
= 1000 millimeters (mm)
= 0.001 kilometers (km)
= 3.281 feet (ft)
= 1.094 yard (yd)
= 39.370 inches (in)

and, 5280 ft = 1 mile (mi)

Time is measured in terms of an arbitrarily defined SI unit called the second (s). One second was originally defined as 1/86,400 of a mean solar day. It is now defined in terms of the vibration frequency of the nucleons within a particular atomic nucleus.

LABORATORY

The student should be able to use various length measuring devices (ruler, meter stick, vernier caliper, micrometer caliper, cathetometer, measuring microscope, etc.) and thus measure distances, areas, and volumes.

When making calculations, it is important to consider significant figures. A significant figure is defined as one that is known to be reasonably trustworthy. One and only one estimated or doubtful figure is retained and regarded as significant in reading a physical measurement. A useful, but very rough, rule states that in multiplication and division the result should have as many significant figures as the least accurate of the factors. All integer data (e.g., 5 s, 4 m, 40 days, 100 years, 4000 miles, etc.) will be considered to have 3 significant figures unless otherwise stated.

WORKED EXAMPLES

1. The length of a football field is 100.0 yards. Express this length in meters.

$$100.0 \text{ yd} = (100.0 \text{ yd})\left(\frac{1 \text{ m}}{1.094 \text{ yd}}\right) = 91.41 \text{ m}$$

*SI is the International System of units which is the preferred system of units for scientific measurement.

2. One of the Olympic track events is the 100.0 meter dash. Express this distance in feet.

$$100.0 \text{ meters} = (100.0 \text{ m})\left(\frac{3.281 \text{ ft}}{1 \text{ m}}\right) = 328.1 \text{ ft}$$

3. The distance between St. Louis, Missouri and Chicago, Illinois is about 300 miles. Express this distance in kilometers.

$$300 \text{ mi} = (300 \text{ mi})\left(\frac{5280 \text{ ft}}{\text{mi}}\right)\left(\frac{1 \text{ m}}{3.281 \text{ ft}}\right)\left(\frac{0.001 \text{ km}}{1 \text{ m}}\right)$$

$$= 483 \text{ km}$$

STUDENT PROBLEMS

1. The distance between the sun and the earth is about 93,000,000 miles. Express this distance in kilometers.

   (1.50×10^8 km)

2. The diameter of a typical carnival ferris wheel is 10.0 meters. How far (in feet) will a rider travel in one revolution of the ferris wheel? HINT: The circumference of a circle of diameter D is πD.

   (103 ft)

3. An "Olympic sized" swimming pool is at least 25.0 meters long. Many American pools are 25.0 yards long. Which pool is longer? By how much?

   (Olympic sized, 2.35 yd)

4. An American made automobile typically has a speedometer with a range of 0 to 120 mph. In the not too distant future the United States will probably convert to the metric system and speedometers will be calibrated in kilometers per hour. What range of numbers will then appear on the speedometer scale?

   (0 to 193 km/hr)

SECTION 2 - MASS AND WEIGHT

Mass may be thought of as a quantitative measure of the resistance of a body to a change in its state of motion. For example, a more massive body is more difficult to start from rest or bring to a stop than is a less massive body (frictional effects are ignored).

In the metric system mass is measured in an arbitrarily defined SI unit called the kilogram (kg) where

$$1 \text{ kilogram (kg)} = 1000 \text{ grams (g)}$$

In the English system mass is measured in slugs (sl)

Mass may also be thought of as a measure of the quantity of material in a body.

Weight measures the pull of gravity on an object.

In the metric system weight is measured in a unit called the newton (N).

Weight in newtons (on earth) =(9.8) (mass in kg)

Thus, 1 kilogram weighs (on earth) 9.8 Newtons.

In the English system weight is measured in pounds (lb); 16 ounces (oz)= 1 lb.

Weight in pounds (on earth) = (32) (mass in sl)

A useful conversion factor is

$$1 \text{ lb} = 4.45 \text{ N}$$

Mass should not be confused with weight. Mass and weight are proportional to each other but mass and weight measure two basically different quantities. For example, on earth a mass of 1 kg weighs about 2.2 pounds. Similarly, a mass of 2 kg weighs about 4.4 pounds. These same masses would have different weights if they were taken to the moon or any other celestial body with a different size and/or composition than the earth.

Mass is a more fundamental quantity than weight since the mass of an object is independent of its location. Weight, on the other hand, varies with location. For example, the weight of a person on the moon is only about 1/6 of his weight on the earth but his mass is the same. A "150 lb earthman" weighs only about 25 lb on the moon.

Mass may also be defined operationally. If two isolated objects are made to interact with each other (for example, two gliders on a level air track are tied together by a compressed spring which is subsequently released) and as a result of the interaction object #1 travels a distance of s_1, while object #2 travels a distance of s_2, then, by definition

$$\frac{s_1}{s_2} = \frac{\text{mass } 2}{\text{mass } 1} = \frac{m_2}{m_1}$$

For example, if as a result of an interaction one object travels 40 cm while the second object travels 15 cm the ratio of mass #2 to mass #1 is (40 cm)/(15 cm), or 2.67.

If object #1 is chosen to be equal to one unit of mass, then

$$\text{(mass \#2)} = m_2 = \frac{s_1}{s_2}\text{ (mass \#1)} = \frac{s_1}{s_2}\text{ (1 mass unit)} = \frac{s_1}{s_2}$$

For example, if in the above example mass #1 is 1 kg, then $m_2 = 2.67$ kg.

LABORATORY

The student should be able to use various types of balances (spring balance, double pan balance, bathroom scale, substitution type balance, etc.) and thus measure a range of masses (several kilograms to 10^{-4} - 10^{-5} grams).

Also, using a device such as the air track, the student should be able to operationally define and measure mass.

WORKED EXAMPLES

1. In a grocery store meat is sold by the pound (one pound is the standard weight of a certain mass). A shopper purchases a 5 pound roast. Suppose the scale were calibrated in kilograms. What reading would appear on the grocer's scale?

$$5 \text{ lb} = (5 \text{ lb})\left(\frac{1 \text{ kg}}{2.2 \text{ lb}}\right) = 2.3 \text{ kg}$$

(NOTE: Strictly speaking 1 kg $\neq$ 2.2 lb, but 1 kg weighs 2.2 lb.)

2. A typical adult female weighs 110 lb. Express this weight in newtons.

$$110 \text{ lb} = (110 \text{ lb})\left(\frac{1 \text{ kg}}{2.2 \text{ lb}}\right)\left(\frac{9.8 \text{ N}}{1 \text{ kg}}\right) = 490 \text{ N}$$

$$= 4.9 \times 10^2 \text{ N}$$

3. What is the weight of a 40 kg mass on earth?

$$W = (40 \text{ kg})\left(\frac{9.8 \text{ N}}{\text{kg}}\right) = 390 \text{ N} = 3.9 \times 10^2 \text{ N}$$

STUDENT PROBLEMS

1. In an attempt to measure the unknown mass of an air track glider by having it interact with a standard 0.25 kg glider on a level air track the following data was recorded:

distance moved by standard glider = 40 cm
distance moved by unknown mass = 18 cm

Find the mass of the unknown glider.

(0.56 kg)

2. The weight of a certain person is 150 lb. Find this mass in kilograms.

(68 kg)

3. A can of pears weighs 12 3/4 ounces. Express this weight in newtons.

(3.6 N)

SECTION 3 - MEASUREMENT AND ERROR; ACCURACY; PRECISION

Whenever you make a measurement, there are certain errors in the measurements.

A) Systematic Error. These errors are produced by known defects in the measuring instruments or their use. They can be eliminated from the final results of measurements.

B) Random Errors arise from totally unpredictable sources and cannot be eliminated. However, their effect on the final results of measurements can be minimized.

The Arithmetic Mean of a set of measurements $m_1, m_2, m_3, m_4, \ldots m_N$ of the same quantity is given by

$$\text{ARITHMETIC MEAN} = \frac{\text{sum of readings}}{\text{number of readings}}$$

$$= \frac{1}{N} \sum_{i=1}^{N} m_i$$

$$= \bar{m}$$

For example, if a length is measured 7 times and the results (in cm) are 4.52, 4.64, 4.72, 4.54, 4.63, 4.57 and 4.59 the arithmetic mean is

$$\bar{m} = \frac{4.52 + 4.64 + 4.72 + 4.54 + 4.63 + 4.57 + 4.59}{7} \text{ cm}$$

$$= \frac{32.21}{7} = 4.60 \text{ cm}$$

The absolute deviation from the mean d_i of an individual reading m_i is the absolute value of the difference between the reading m_i and the mean value $\bar{m}$ of the set of readings.

$$d_i = |m_i - \bar{m}|$$

For example, for the data above, the absolute deviations from the mean are 0.08, 0.04, 0.12, 0.06, 0.03, 0.03 and 0.01 cm, respectively.

The average absolute deviation is defined by

$$\bar{d} = \frac{\text{sum of absolute deviations from the mean}}{\text{number of readings}}$$

$$= \frac{\sum_{i=1}^{N} d_i}{N}$$

For example, for the above set of readings

$$\bar{d} = \frac{0.08 + 0.04 + 0.12 + 0.06 + 0.03 + 0.03 + 0.01}{7} \text{ cm}$$

$$= \frac{0.37}{7} \text{ cm}$$

$$= 0.05 \text{ cm}$$

The uncertainty in a set of measurements is usually considered to be $\pm\bar{d}$.

For example, for the above set of data the uncertainty is ±0.05 cm.

The measured value of a quantity is usually listed as

$$\bar{m} \pm \bar{d}$$

This means that the "true value" probably lies in the range defined by $(\bar{m} - \bar{d})$ and $(\bar{m} + \bar{d})$.

For example, in the data used above the results of the measurement would be recorded as

$$(4.60 \pm 0.05) \text{ cm}$$

This means that the "true reading" probably lies in the range 4.55 to 4.65 cm.

The results of different sets of measurements of the same quantity or of sets of measurements of different quantities are considered to be identical if the ranges defined by $\bar{m} \pm \bar{d}$ overlap.

For example, if the speed of a car were measured on 3 separate occasions and the results were (2.50 ± 0.03) m/s; (2.56 ± 0.01) m/s and (2.45 ± 0.05) m/s, one might say that the results of the first and third measurements were identical since the ranges defined by ($\overline{m}$ ± uncertainty) overlap. The range indicated in the second measurement does not overlap either of the other two ranges; hence, the second measurement is significantly different than the other two measurements.

The precision of a set of measurements refers to the size of the uncertainty. A small uncertainty indicates a precise measurement. Precision implies repeatability.

For example, the second speed measurement above is the most precisely determined value.

If all measurements are alike, one-half of the smallest scale division on the instrument is the uncertainty. This is also the minimum uncertainty for any measurement.

Accuracy refers to the correctness of a measurement. A measurement may be precise but inaccurate. For example, a precise measurement may be made on an instrument which has a systematic error (e.g., a "non-zeroed" scale). An accurate measuring device is one which gives the same results as an instrument which is correctly calibrated against a standard.

LABORATORY

The student should be able to apply the above techniques to the determination of the "best values" of a variety of experimentally determined quantities. For example,

Value of a resistance
Diameter of a cylinder
Mass of a typical penny
Speed of a glider in a given environment on an air track
Average velocity of a falling body
etc.

WORKED EXAMPLES

1. Repeated measurements of the acceleration of a falling body yield the following results: 9.75, 9.73, 9.84, 9.87, 9.99, 9.90, 9.69, 9.79, 9.85 and 9.80 m/s^2. Find $\overline{m}$, $\overline{d}$.

m_i	d_i
9.75	0.07
9.73	0.09
9.84	0.02
9.87	0.05
9.99	0.17
9.90	0.08
9.69	0.13
9.79	0.03
9.85	0.03
9.80	0.02
10 | 98.21	10 | 0.69
$\overline{m} = 9.82$	$\overline{d} = 0.07$

Therefore, result is $(9.82 \pm 0.07)\ m/s^2$.

2. According the Life Insurance Statistics the "normal life expectancy" for a white female born in the United States today (1975) is about 74-75 years. Just what is exactly meant by this statement?

It is really somewhat hard to interpret this statement. What it may mean is that if all white females born this year in the United States were able to live their entire lives under the present social and medical conditions then the average age at death would be 74-75 years.

The statement above does not mean that all white females born in 1975 will live to be 74-75; it merely implies that the "average" one will live to be 74-75.

The statement above says nothing about the uncertainty in the 74-75 figures. For example, the case of most dying in the range of 70-80 as well as the case where large numbers would die of various childhood diseases and complications of childbirth and large numbers would live beyond 80 years could both lead to the 74-75 average.

Hence, the statement above is incomplete and, therefore, somewhat meaningless. It is a good example of the misuse of statistical data.

How could the above statement be improved?

STUDENT PROBLEMS

1. Electrical resistors are sometimes labeled as 1%, 5%, 10% or 20% resistors. What do you think this terminology means? In particular, explain this terminology as it applies to a 10%, 500 ohm resistor.

   (refers to uncertainty; (500 ± 50) ohms)

2. The "normal score" on an "IQ" test is in the range 90-110. What is meant by a "normal score"?

3. When a person buys a life insurance policy he is essentially betting the company that he will die before the company has collected enough money from him to offset the loss it incurs when he dies. Discuss in a general way how the insurance company sets the rates for its various life insurance policies.

4. Repeated measurements of the time of fall of a freely falling object yield the following results: 4.65, 4.86, 4.67, 4.76, 4.84, 4.84, 4.94, 4.66, 4.90, 4.70, 4.95, 4.65, 4.91, 4.74, 4.99 s. Find $\bar{m}$, and $\bar{d}$.

   (4.80 s; 0.11 s)

SECTION 4 - BASIC ELECTRICAL CONCEPTS*

Electric charge is a property of matter which causes objects possessing this characteristic to react with each other in certain ways. In particular, there are two types of electric charge - positive and negative. Objects having charges of the same type repel each other and objects having charges of different types attract each other.

The unit of electrical charge is the coulomb (C).

Current refers to a flow of electrically charged particles. In many cases of interest this flow is restricted to a wire. Here negatively charged particles called electrons flow in the wire. If this flow is always in one direction we speak of a direct current (DC). If the flow periodically reverses itself, first flowing one way and then the other way, we speak of an alternating current (AC).

*This material is presented here for reference purposes and should not really be considered as part of Chapter I. However, much of this material will be quite useful to help students understand Chapter IV - Temperature and Heat - where much of the material (laboratory measurements, in particular) is related to electrical concepts.

Current is measured in amperes (A). If 1 coulomb of electric charge passes a point in 1 second, we say that the current is 1 ampere - i.e., 1 A = 1 C/s.

A meter used to measure current is called an ammeter. The electrical symbol for a DC ammeter is

The electron has a charge of 1.6×10^{-19} C. Hence, one ampere means that 6.25×10^{18} electrons flow past a point each second when the current is 1 amp.

Electric circuits are combinations of devices through which an electric current flows. For a steady current to flow the circuit must provide a path(s) which is(are) complete (i.e., closed).

The Potential Difference (Voltage) V between 2 points in a circuit is measured by the work W (in joules) required to move a unit positive charge from one point to the other.

Voltage is measured in volts (V). If one joule of work is needed to move 1 coulomb of charge between two points in a circuit, we say that the potential difference between these two points is 1 volt. That is, 1 V = 1 J/C.

If a charge Q moves between 2 points in a circuit that differ in potential by V volts, then the work W done by the charge Q in moving from the higher to the lower potential point is W = QV.

A meter used to measure voltage is called a voltmeter. The electrical symbol for a DC voltmeter is

Electromotive Force (EMF) is a term which describes a device which does work on the charged particles moving through it. A battery or a generator are examples of sources of EMF. In moving through a source of EMF a positively charged particle picks up electrical energy as it goes from the lower to the higher potential side of the source of EMF. EMF is measured in volts. The EMF is measured between the terminals of the device when it is delivering no current.

For example, an ordinary "D-cell" used in flashlights has an EMF of 1.5 V.

EMF's deliver currents to external circuits connected to them. The electrical symbol for a battery is 

and for a DC generator is 

The <u>resistance R</u> of a conductor measures the impeding effect it has on a current passing through it. Resistance is measured in <u>ohms</u> (Ω). An ohm is defined as the resistance of a conductor which will carry 1 ampere when the ends of the conductor differ in potential by 1 volt. That is,

$$1\ \Omega = 1\ \mathrm{V/A}$$

Resistance also depends upon the length ℓ (in meters), cross-sectional area A (in m^2), material and temperature of the conductor:

$$R = \rho \frac{\ell}{A}$$

where ρ is the <u>resistivity</u> - a characteristic of the material. ρ is measured in Ωm. Resistivity is a function of temperature.

The symbol for resistance is 

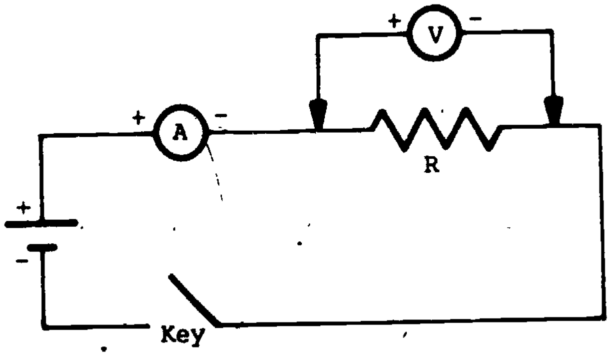
<u>Ohm's Law</u> is a relationship holding for most metallic conductors:

$$I = \frac{V}{R}$$

This relationship can be applied to any part of a circuit or to the entire circuit.

For example, if a voltage of 5 V is applied to a 10 Ω resistor, the current will be $I = V/R = 5\ \mathrm{V}/10\ \Omega = 0.500$ A.

Resistance can be measured by using an ammeter and a voltmeter connected as follows:



Be careful to observe polarities when connecting meters. The key is merely a device for turning the current off and on.

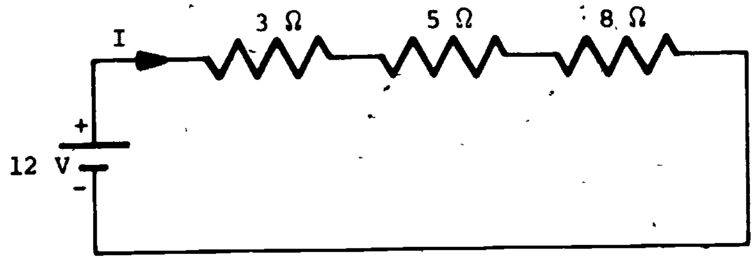
Voltages across a resistor are called <u>voltage drops (IR drops)</u>, using the convention that current goes through a resistor from the high potential end to the low potential end.

In a series circuit all of the current flows through each resistor successively. The current in all of the resistors is identical and the total potential difference across resistors connected in series is equal to the sum of the potential differences across the separate resistors. The total equivalent resistance R of a series combination R_1, R_2, R_3,...is

$$R = R_1 + R_2 + R_3 + \ldots ,$$

For example, for 3 resistors - 3 Ω, 5 Ω, 8 Ω - connected in series $R = (3 + 5 + 8)\Omega = 16.0\ \Omega$.

A typical series circuit, using these resistors, might be the following:



In a parallel circuit the current can follow two or more separate paths. The total current through the parallel combination is equal to the sum of the separate currents in the individual branches of the combination. The voltage is the same across each of the branches. The total equivalent resistance R of a parallel combination R_1, R_2, R_3, ... is

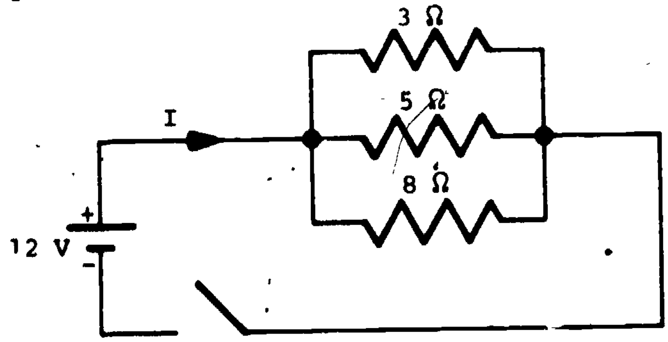
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

For example, for 3 resistors - 3 Ω, 5 Ω, 8 Ω - connected in parallel

$$\frac{1}{R} = \frac{1}{3\ \Omega} + \frac{1}{5\ \Omega} + \frac{1}{8\ \Omega} = \frac{40 + 24 + 15}{120\ \Omega} = \frac{79}{120\ \Omega}$$

$$R = \frac{120\ \Omega}{79} = 1.52\ \Omega$$

A typical parallel circuit, using these resistors, might be



Electrical Energy U is measured in joules (J) and kilowatt-hours (kW-hr). 1 kW-hr = 3.6×10^6 J. For a DC-circuit (and certain AC circuits) obeying Ohm's Law

$$U = IVt = I^2Rt = \frac{V^2}{R}t$$

For example, if a current of 5 A flows through a 10 Ω resistor for 5 minutes, the electrical energy developed is $U = I^2Rt = (5\ \text{A})^2(10\ \Omega)(300\ \text{s}) = 7.50 \times 10^4\ \text{J} = (7.50 \times 10^4\ \text{J})(1\ \text{kW-hr}/3.6 \times 10^6\ \text{J}) = 0.208$ kW-hr.

Electrical Power P is electrical (energy/time) and is measured in watts (W) and kilowatts (kW); 1 kW = 1000 W. In a DC-circuit (and in some AC circuits) for which Ohm's Law is valid

$$P = IV = I^2R = \frac{V^2}{R}$$

For example, if a direct current of 5 A flows through a 10 Ω resistor, the electrical power developed is $P = I^2R = (5\ \text{A})^2(10\ \Omega) = 250$ W.

Also, 1 watt = 1 joule/second or 1 W = 1 J/s.

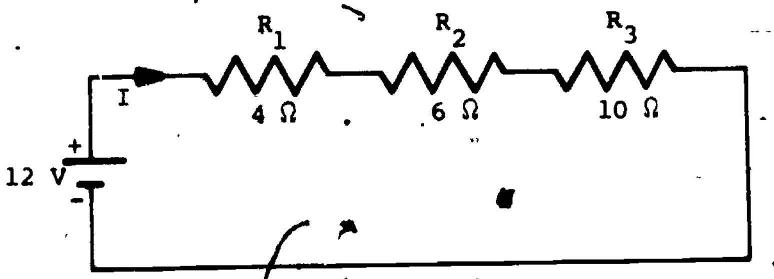
In various electric circuits electrical energy is converted into heat (toaster, iron, space heater, etc.), sound (loudspeaker in stereos, TV's, etc.), motion (various kinds of motors), light (incandescent lamp).

LABORATORY

Given a circuit diagram, a battery, connecting wires, resistances, ammeter, voltmeter, timer, etc., the student should be able to hook up the circuit and measure the various electrical quantities - current, voltage, power, energy, - applicable to all parts of the circuit.

SOLVED PROBLEMS

1. Find the current in the circuit below. Also, find the electrical energy consumed by the circuit in one hour.



The total resistance R of this series circuit is

$$R = R_1 + R_2 + R_3 = 4\ \Omega + 6\ \Omega + 10\ \Omega = 20.0\ \Omega$$

Therefore, the current flowing is

$$I = \frac{V}{R} = \frac{12\ V}{20.0\ \Omega} = 0.600\ A$$

The electrical energy U consumed is

$$U = Pt = IVt = I^2Rt = \frac{V^2}{R}t$$

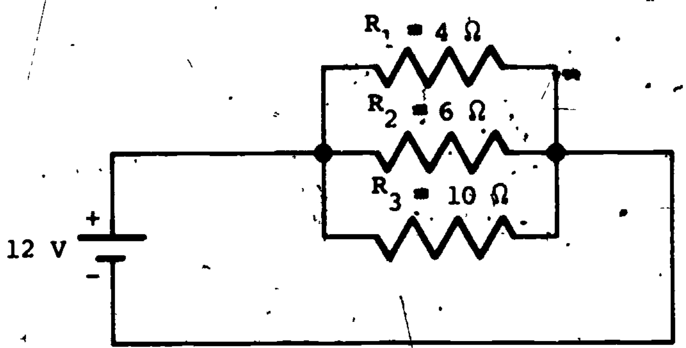
$$IVt = (0.600\ A)(12\ V)(3600\ s) = 2.59 \times 10^4\ J$$

$$I^2Rt = (0.600\ A)^2(20\ \Omega)(3600\ s) = 2.59 \times 10^4\ J$$

$$\frac{V^2}{R}t = \frac{(12\ V)^2}{20\ \Omega}(3600\ s) = (2.59 \times 10^4\ J)\left(\frac{1\ kW\text{-}hr}{3.6 \times 10^6\ J}\right)$$

$$= 0.0072\ kW\text{-}hr$$

2. Find the current in the circuit below. Also, find the electrical energy consumed by the 10 Ω resistor in one hour.



The total resistance R of the circuit is

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{4\ \Omega} + \frac{1}{6\ \Omega} + \frac{1}{10\ \Omega} = \frac{15 + 10 + 6}{60\ \Omega} = \frac{31}{60}\ \Omega$$

$$R = \frac{60}{31}\ \Omega = 1.94\ \Omega$$

$$I = \frac{V}{R} = \frac{12\ V}{1.94\ \Omega} = 6.20\ A$$

The 10 Ω resistor has 12 V across it (as do the 4 Ω and 6 Ω resistors).

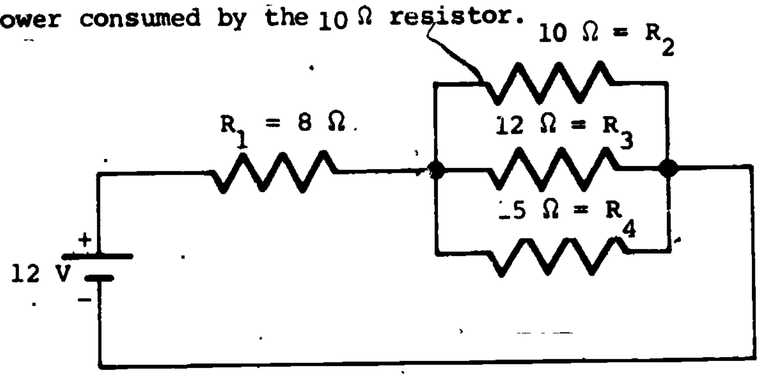
$$U = Pt = \frac{V^2}{R}t = \left(\frac{(12\ V)^2}{10\ \Omega}\right)(3600\ s) = (1.44 \times 10^{-2}\ kW)(1\ hr)$$

$$= 1.44 \times 10^{-2}\ kW\text{-}hr$$

$$= (1.44 \times 10^{-2}\ kW\text{-}hr)\left(\frac{3.6 \times 10^6\ J}{kW\text{-}hr}\right)$$

$$= 5.2 \times 10^4\ J$$

3. In the circuit below, find the current in the 8 Ω resistor and the power consumed by the 10 Ω resistor.



The resistance R_p of the parallel combination is

$$\frac{1}{R_p} = \frac{1}{10\ \Omega} + \frac{1}{12\ \Omega} + \frac{1}{15\ \Omega} = \frac{12 + 10 + 8}{120\ \Omega} = \frac{30}{120\ \Omega}$$

$$R_p = \frac{120}{30}\ \Omega = 4\ \Omega = 4.00\ \Omega$$

The total resistance R of the circuit is

$$R = 8\ \Omega + 4\ \Omega = 12.0\ \Omega$$

The current I delivered by the battery is

$$I = \frac{V}{R} = \frac{12\ V}{12\ \Omega} = 1.00\ A$$

1.00 A flows through the 8 Ω resistor. Hence the potential drop across this resistor is

$$(1.00\ A)(8.00\ \Omega) = 8.00\ V$$

Therefore, the voltage across the 10 Ω resistor is (12 − 8)V = 4.00 V and the power P used by this resistor is

$$P = \frac{V^2}{R} = \frac{(4\ V)^2}{10\ \Omega} = 1.60\ W$$

STUDENT PROBLEMS

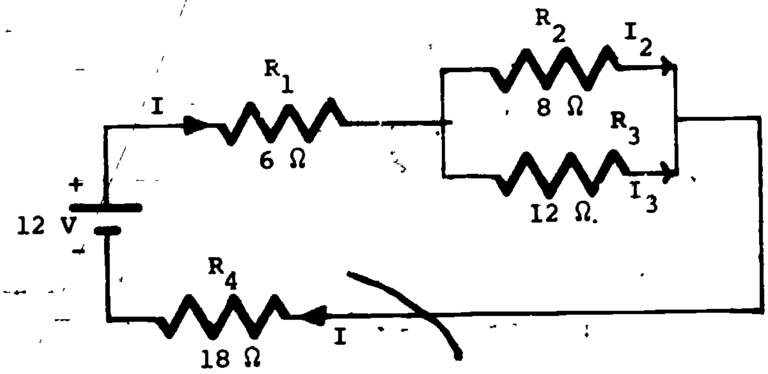
1. What resistance must be placed in parallel with a 30 Ω resistor in order to reduce the combined resistance to 10 Ω?

(15.0 Ω)

2. Compute the resistance of 500 m of copper wire (resistivity is 1.8×10^{-8} Ωm) having a cross-section of 0.30 mm^2.

(30 Ω)

3. For the circuit below find all of the currents and voltage drops. How much electrical energy is consumed by the circuit in 30 minutes?



($I = 0.417$ A
$I_2 = 0.250$ A
$I_3 = 0.167$ A
Voltage across $R_1 = 2.50$ V
Across $(R_2 + R_3) = 2.00$ V
Across $R_4 = 7.50$ V ; 9.0×10^3 J)

4. A 5 Ω electric heater operates on a 110 V line. Find the rate at which heat is developed.

(2.42×10^3 W)

CHAPTER II

TRANSLATIONAL MOTION

SECTION 1 - DISTANCE

The position P of an object in space can be specified in terms of its X-, Y- and Z- coordinates: (X,Y,Z).

For example, the position of Point P in the XY plane, as shown below, could be specified as (3,4); i.e., 3 units right of 0 on X and 4 units above 0 on Y.

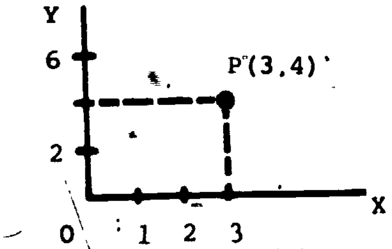


Fig. (1)

The distance s between two points (X_1, Y_1, Z_1) and (X_2, Y_2, Z_2) is given by

$$s = \sqrt{(X_2-X_1)^2 + (Y_2-Y_1)^2 + (Z_2-Z_1)^2}$$

For example, if an object P moves along a straight line from point (1,2,3) to point (4,3,7), the total distance s traveled is

$$s = \sqrt{(4-1)^2 + (3-2)^2 + (7-3)^2}$$

$$= \sqrt{3^2 + 1^2 + 4^2}$$

$$= \sqrt{26}$$

$$= 5.10$$

GRAPHICAL REPRESENTATION

The position of an object may or may not change with time. This movement (or non-movement) may be represented on a position-time plot, time plotted horizontally.

For example, Figure (2) below represents the motion of a stationary object while Figure (3) represents the motion of a moving object.

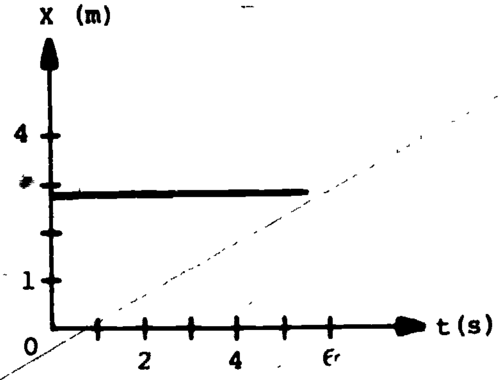


Fig. (2)

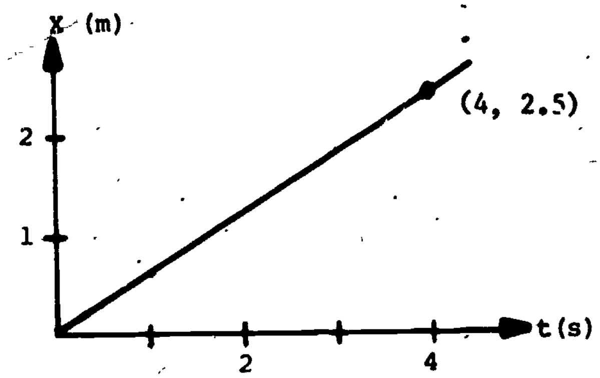


Fig. (3)

WORKED EXAMPLES

1. An automobile travels from St. Louis, Missouri to Chicago, Illinois, a distance of approximately 300 miles. Find this distance in meters.

$$300 \text{ miles} = (300 \text{ mi})\left(\frac{5280 \text{ ft}}{1 \text{ mile}}\right)\left(\frac{1 \text{ m}}{3.281 \text{ ft}}\right) = 4.83 \times 10^5 \text{m}$$

2. Graphically interpret the motion represented by the following position-time plot.

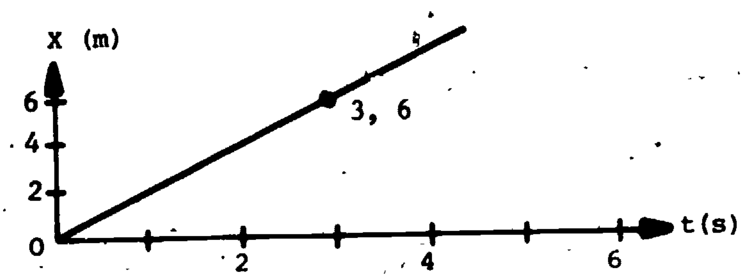


Fig. (4)

at t = 0, X = 0
= 1s, X = 2m
= 2s, X = 4m
= 3s, X = 6m

CONCLUSION: the object travels 2m during every second of time; i.e, it moves uniformly

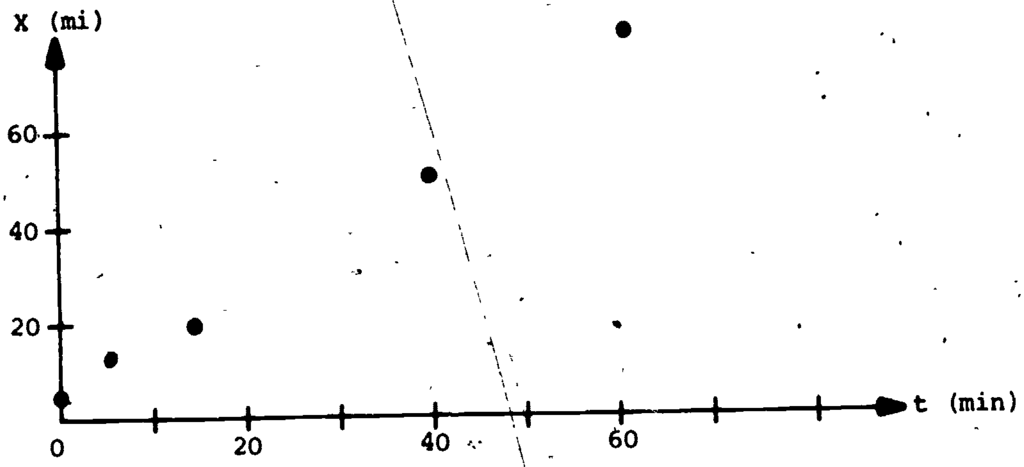
STUDENT PROBLEMS

1. A typical carnival carousel has a radius of 10.00 meters. On a typical "ride" the carousel revolves 50.00 times. What total distance is covered by a rider during one ride?

(3142 m)

2. A car is traveling down an interstate highway and the driver notices that he passes the "5 mile post" at 3:00 p.m., the "10 mile post" at 3:05 p.m., the "20 mile post" at 3:14 p.m., the "50 mile post" at 3:40 p.m. and the "75 mile post" at 4:00 p.m. Graphically represent this motion on a time-position plot, time plotted horizontally. Describe the general characteristics of this motion.

Ans.



(The motion is not uniform; there is a gradual increase in the speed of the car.)

SECTION 2 - SPEED AND VELOCITY

The average speed ($\overline{V}$) of a body which travels a distance s in time t is defined to be

$$\overline{V} = \frac{s}{t}$$

Speed merely specifies the numerical value of the rate of motion of an object. It is a scalar quantity.

The units of speed are the units of length divided by the units of time. For example, speed may be measured in meters per second (m/s), feet per second (ft/s), miles per hour (mph or mi/hr), etc.

For example, the average speed of an automobile which takes 6 hours to go from St. Louis, Missouri to Chicago, Illinois, a distance of 300 miles is

$$\bar{V} = \frac{s}{t} = \frac{300 \text{ mi}}{6 \text{ hr}} = 50.0 \frac{\text{mi}}{\text{hr}} = 50.0 \text{ mph}$$

<u>NOTE</u>: 60 mph ≡ 88 ft/s is a useful conversion factor.

<u>Velocity</u> is a vector quantity, the magnitude of which is equal to the speed. The direction of the velocity is the direction of the motion. The units of velocity are the same as the units of speed; however, a direction must be appended to each velocity reading: for example, 30 m/s, North.

Velocity changes if speed and/or direction change(s).

For motion along a straight line speed and velocity are synonymous if one employs the additional convention that motion in one direction along the line has a positive velocity while motion along the other direction has a negative velocity. Usually, the directions of the conventional coordinate axes define the positive and negative directions.

Speed and/or velocity can be represented graphically. For example, Fig. (5) represents motion at a constant speed while Fig. (6) represents motion at a steadily increasing speed.

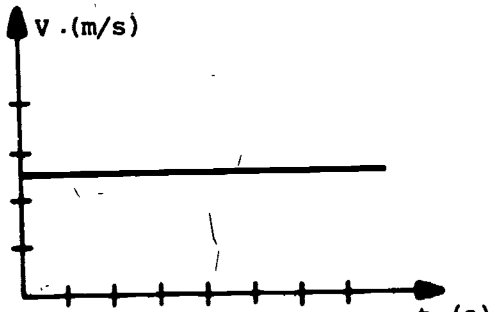


Fig. (5)

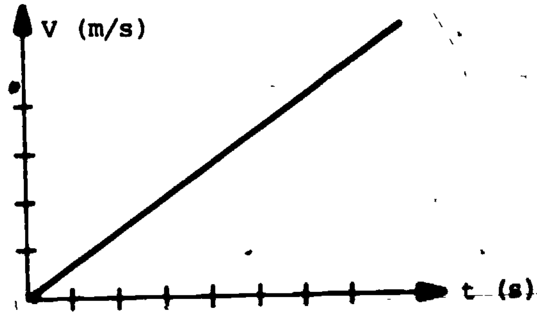


Fig. (6)

More complex types of motion are, of course, possible.

<u>Speed may also be interpreted as the slope of the position-time curve.</u>

$$\text{Slope} = \frac{\text{rise of curve}}{\text{run of curve}}$$

For example, in Fig. (3) for a run of 4 seconds ($t = 0$ to $t = 4$ s), the corresponding rise is 2.5 meters ($X = 0$ to $X = 2.5$ m). Hence,

$$\text{speed} = \frac{\text{rise}}{\text{run}} = \frac{2.5 \text{ m} - 0}{4.00 \text{ s} - 0} = \frac{2.5 \text{ m}}{4.00 \text{ s}} = 0.63 \frac{\text{m}}{\text{s}}$$

Therefore, Fig. (3) represents a uniform motion with a speed of 0.63 m/s.

For non-linear curves, slope can be defined at each point, referring to the tangent line at that point. The result obtained should be interpreted as the <u>instantaneous</u> speed or velocity at that point.

The motions represented by Fig. (7) below are motions which do not have graphs with single values for the slope. That is, the slopes of Fig. (7) change with time. For example, in Fig. (7b) the slope steadily increases with time; this indicates a steadily increasing speed. In Fig. (7a) the slope initially increases with time but then it levels off and attains a constant value.

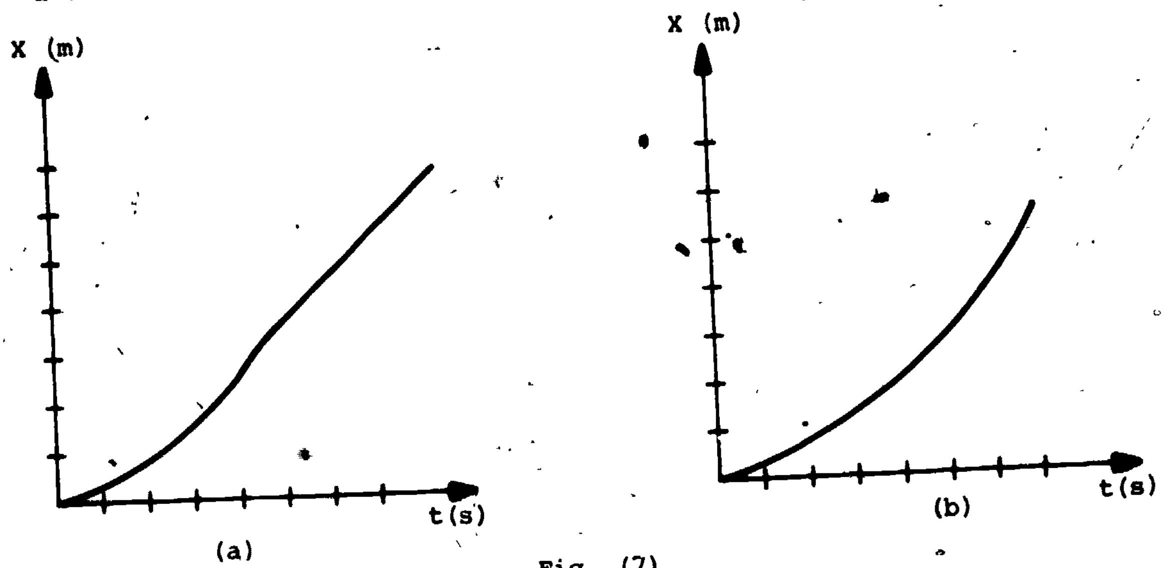


Fig. (7)

LABORATORY

The student, using such length measuring devices as meter sticks and such time measuring devices as electric timers and stroboscopes, should be able to measure the average speeds of such objects as falling bodies or air track gliders.

WORKED EXAMPLES

1. A bus travels 250 miles in 5 hours. Find the average speed in miles per hour and feet per second.

$$\bar{V} = \frac{s}{t} = \frac{250 \text{ mi}}{5 \text{ hr}} = 50.0 \text{ mph}$$

$$= (50.0 \frac{\text{mi}}{\text{hr}})(\frac{88 \text{ ft/s}}{60 \text{ mph}})$$

$$= 73.3 \frac{\text{ft}}{\text{s}}$$

2. An automobile is traveling at a speed of 60 mph when the driver sees a dog dart onto the road ahead and stop dead in its tracks. Assuming a reaction time of 1.50 seconds, how far (in meters) will the car travel while the driver moves his foot from the accelerator to the brake?

$$\bar{v} = \frac{s}{t}$$

$$s = \bar{v}t = (60.0\ \frac{mi}{hr})(1.50\ s)$$

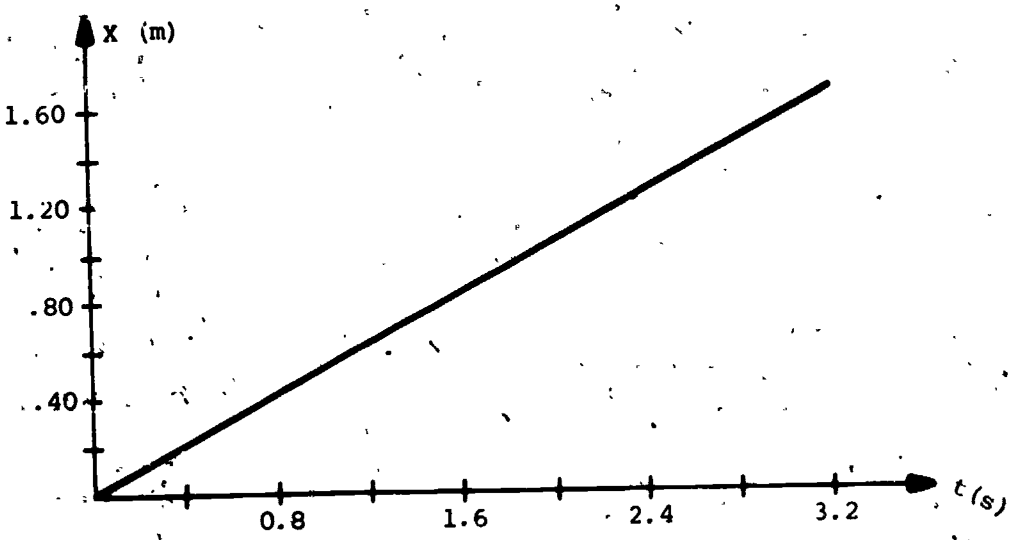
$$= (60.0\ \frac{mi}{hr})\frac{(88\ ft/s)}{(60\ mph)}(\frac{1\ m}{3.281\ ft})(1.50\ s)$$

$$= 40.2\ m$$

3. The following data are recorded by a group of students studying the motion of an isolated moving glider on a frictionless level air track.

POSITION OF GLIDER (x) (m)	TIME OF OBSERVATION (t) (s)
0	0
0.20	0.4
0.40	0.8
0.60	1.2
0.80	1.6
1.00	2.0
1.20	2.4
1.40	2.8
1.60	3.2

Construct a position-time plot for this data. From this plot determine the average speed of the glider.



$$\vec{V} = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{(1.60\ \text{m} - 0\ \text{m})}{(3.2\ \text{s} - 0\ \text{s})} = \frac{1.60\ \text{m}}{3.2\ \text{s}} = 0.50\ \frac{\text{m}}{\text{s}}$$

STUDENT PROBLEMS

1. A bus travels 75 kilometers in 5 hours. Find its speed in km/hr.

   (15.0 km/hr)

2. A driver for a trucking company is to deliver a load of scrap iron to a suburban New York factory and then return to his suburban Washington D.C. home base with his empty truck. Assuming that the distance between the two locations is 100 miles, that the unloading process takes two hours, that the driver stops for a half an hour along the road for lunch and that he returns to his home base at 5:00 p.m. (he left at 8:00 a.m.), what average speed must he maintain on the road?

   (30.8 mph)

3. Johnny Rutherford, winner of the 1974 Indianapolis 500 race maintained an average speed of 158.6 mph. How long did the race take (4 significant figures)?

   (3.153 hr)

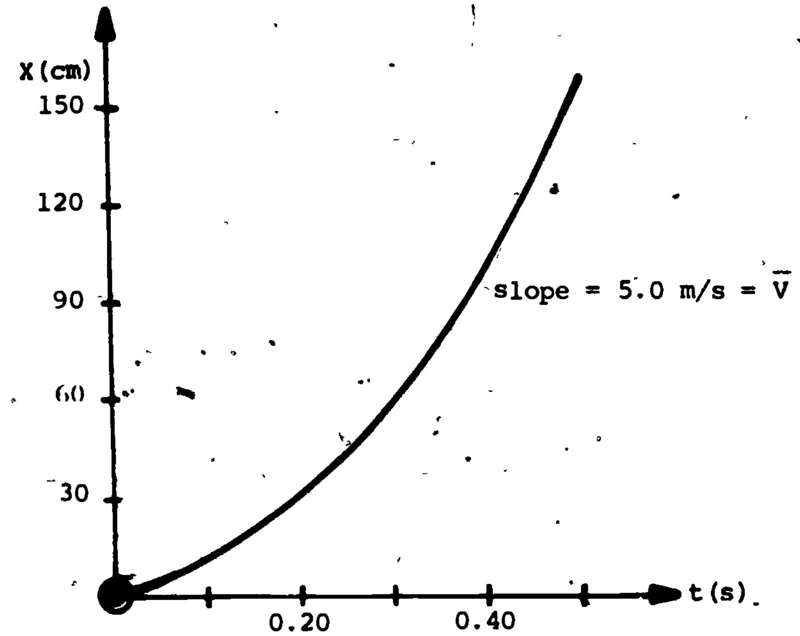
4. Light travels at a speed of about 300,000 km/s in a vacuum. The nearest star, Proxima Centauri is 4.2 light years away. (A light year is the distance that light will travel in one year.) What is the distance to Proxima Centauri?

   (4.0×10^{13} km)

5. A ball was dropped from rest and its position was recorded at intervals of 0.05 s. The following data were recorded:

POSITION (X) (cm)	TIME (t) (s)
0.0	0.0
4.0	0.05
10.4	0.10
19.3	0.15
30.4	0.20
44.2	0.25
61.4	0.30
73.8	0.35
99.7	0.40
123.1	0.45
148.9	0.50

   Construct a position-time plot. Describe the general characteristics of the motion. Find the slope of the curve towards the end of the motion. What is the average speed towards the end of the motion?



SECTION 3 - ACCELERATION *

Acceleration is a vector quantity which gives the rate at which velocity is changing. For a body having velocity V_o at time t_o and velocity V at time t, the average acceleration $\bar{a}$ is given by

$$\bar{a} = \frac{V - V_o}{t - t_o} = \frac{\Delta V}{\Delta t} .$$

$$V = V_o + \bar{a}t, \text{ if } t_o = 0.$$

ΔV and Δt represent the changes in the velocity and time, respectively.

If the velocity of a body changes uniformly, then we say that its acceleration is constant. In detail, for such a body with velocity V_o at time t_o, and velocity V at time t its constant acceleration a is the same as its average acceleration ($a = \bar{a}$).

$$a = \frac{V - V_o}{t - t_o} = \frac{\Delta V}{\Delta t} \qquad (1)$$

$$V = V_o + at, \text{ if } t_o = 0 \qquad (2)$$

For example, an auto accelerating uniformly from rest to 30 m/s in 10.0 s would have an acceleration of

$$\frac{30.0 \text{ m/s} - 0}{10.0 \text{ s} - 0} = 3.00 \text{ m/s/s} = 3.00 \text{ m/s}^2$$

* This section deals only with motion along a straight line - linear acceleration.

Note that the units of acceleration involve a distance unit divided by the square of a time unit (e.g., ft/s^2, mi/hr^2, m/s^2).

Normally, both time units are kept the same. Mixed time units such as ft/sec/hr, m/s/day are usually not used (mi/hr/s for automobile applications is an exception).

For cases of constant acceleration the average velocity $\overline{V}$ of a body is

$$\overline{V} = \frac{V_o + V}{2} \qquad (3)$$

and the distance s covered in a time t is

$$s = \overline{V}t \qquad (4)$$

which can be shown to give

$$s = V_o t + \frac{1}{2} at^2 \qquad (5)$$

Also, it can be shown that

$$V^2 = V_o^2 + 2\ as \qquad (6)$$

For cases where the body starts from rest, ($V_o = 0$), equations (2), (5) and (6) become

$$V = at;\ s = \frac{1}{2} at^2;\ V^2 = 2\ as,\ (V_o = 0) \qquad (7)$$

For a freely falling body (negligible air resistance) it is found that the acceleration is roughly constant (near the earth's surface). This acceleration is referred to as the acceleration of gravity.

acceleration due to gravity* = $g = 32\ ft/s^2 = 9.8\ m/s^2$

Therefore, for a freely falling body, equations (2), (4), (5), and (6) become

$$V = V_o + gt \qquad (8)$$

$$s = \overline{V}t = \frac{V_o + V}{2} t = V_o t + \frac{1}{2} gt^2 \qquad (9)$$

$$V^2 = V_o^2 + 2\ gs \qquad (10)$$

* We will assume that g has 3 significant figures.

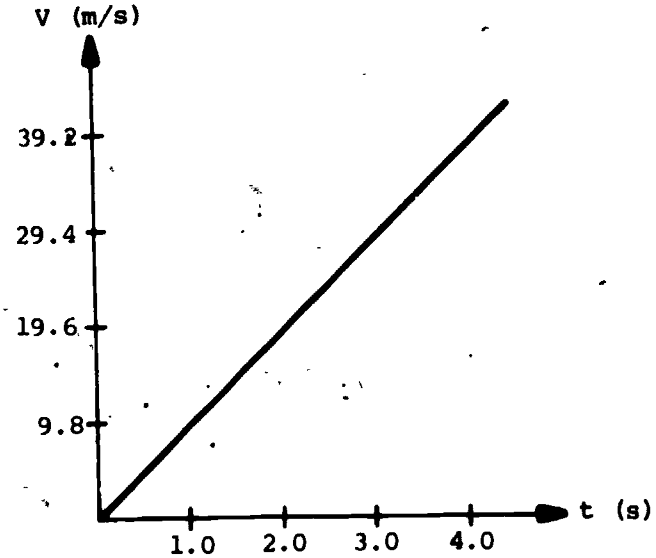
For example, a body dropped from rest will cover 4.90 m in 1 s.

$$s = V_o t + \frac{1}{2} gt^2 = (0)(1\ s) + \frac{1}{2}(9.8\ m/s^2)(1\ s)^2 = 4.90\ m$$

The acceleration of a given motion is equal to the slope of the velocity-time curve of the motion.

For cases of uniform acceleration the velocity changes by the same amount for each uniform increment of time. Hence, the velocity-time curve is a straight line.

In detail, for a body dropped from rest ($V_o = 0$) the velocity-time curve would be as follows:



The slope could be calculated from a run of 3 sec (t = 0 to t = 3 s) and a rise of 29.4 m/s (V = 0 to V = 29.4 m/s).

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{29.4\ m/s - 0}{3\ sec - 0} = 9.80\ m/s^2$$

This, of course, is the acceleration due to gravity, as it should be.

LABORATORY

The student, using such length measuring devices as meter sticks and rulers and such time measuring devices as electric timers, spark timers and stroboscopes, should be able to measure the average speeds and accelerations of such objects as falling bodies and gliders on an air track.

For example, from the following set of data (glider on an air track) the student should be able to construct position-time and velocity-time graphs as well as measure the acceleration.

DATA

POSITION (m)	0	0.11	0.24	0.34	0.56	0.75	0.96	1.19	1.44	1.71
TIME (s)	0	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90

WORKED EXAMPLES

1. A body, starting from rest, moves with a constant acceleration of $10\ m/s^2$ for 6 seconds. Find (a) the speed V at the end of 6 seconds; (b) the average speed $\bar{V}$ for this 5 seconds; and (c) the distance s covered in the 6 seconds.

(a) $V = V_o + at = (0)(6\ s) + (10\ m/s^2)(6\ s) = 60.0\ m/s$

(b) $\bar{V} = \frac{V + V_o}{2} = \frac{60\ m/s + 0}{2} = 30.0\ m/s$

(c) $s = V_o t + \frac{1}{2}at^2 = (0)(6\ s) + \frac{1}{2}(10\ m/s^2)(6\ s)^2 = 180\ m$

or

$s = \bar{V}t = (30\ m/s)(6\ s) = 180\ m$

2. In passing a truck on an upgrade on a highway an automobile accelerates from 30 mph to 60 mph in 20 seconds. Find (a) the acceleration (in ft/s^2); (b) the distance covered (in feet) during the acceleration process. After passing the truck, the auto slows down to 30 mph in 15 seconds. Find (c) the acceleration and (d) the distance covered while the auto is slowing down.

(a) Since 30 mph = 44 ft/s

$$a = \frac{V - V_o}{t} = \frac{88\ ft/s - 44\ ft/s}{20\ s} = \frac{44\ ft/s}{20\ s} = 2.20\ ft/s^2$$

(b) $s = V_o t + \frac{1}{2}at^2 = (44\ ft/s)(20\ s) + \frac{1}{2}(2.20\ ft/s^2)(20\ s)^2$

$= 880\ ft + 440\ ft = 1320\ ft$

$= 1.32 \times 10^3\ ft$

(c) Here the acceleration is negative:

$$a = \frac{V - V_o}{t} = \frac{44\ ft/s - 88\ ft/s}{15\ s} = \frac{-44}{15}\ ft/s = -2.93\ ft/s^2$$

(d) $s = V_ot + \frac{1}{2}at^2 = (88\ ft/s)(15\ s) + \frac{1}{2}(-2.93\ ft/s^2)(15\ s)^2$

$= 1320\ ft - 329.6\ ft$

$= 990.4\ ft$

$= 990\ ft$

3. According to some recent data from the Environmental Protection Agency (E.P.A.) certain automobiles are able to come to a stop from 60 mph in about 250 feet. Find the average, constant acceleration.

$$V = V_o^2 + 2\ as$$

$$a = \frac{V^2 - V_o^2}{2\ s} = \frac{0 - (88\ ft/sec)^2}{2(250\ ft)} = \frac{-7744\ ft^2/s^2}{500\ ft}$$

$$= -15.488\ ft/s^2$$

$$= -15.5\ ft/s^2$$

4. A ball thrown vertically upwards returns to its starting point in 4 seconds. Find its initial speed.

The upwards direction is usually considered to be the positive direction. Hence, here $g = -9.8\ m/s^2$.

At $t = 4\ s$, $s = 0$ (It is back to its starting point.)

$$s = V_ot + \frac{1}{2}at^2$$

$$V_o = \frac{s - \frac{1}{2}at^2}{t} = \frac{0 - \frac{1}{2}(-9.8\ m/s^2)(4\ s)^2}{(4\ s)} = 19.6\ m/s$$

5. A boy leans over the top of a building and throws a ball downwards at a speed of 10 m/s. Find (a) the distance s covered by the ball in 5 seconds; (b) its speed V at the end of the 5 seconds and (c) its average speed $\bar{V}$ over the 5 second period.

Since the motion is downwards and the acceleration is downwards all quantities will have the same sign.

(a) $s = V_ot + \frac{1}{2}at^2 = (10\ m/s)(5\ s) + \frac{1}{2}(9.8\ m/s^2)(5\ s)^2$

$= (50.0 + 122.5)m = 172.5\ m = 173\ m$

(b) $V = V_o + gt = 10 \text{ m/s} + (9.8 \text{ m/s}^2)(5 \text{ s}) = (10 + 49)\frac{\text{m}}{\text{s}} = 59.0 \text{ m/s}$

(c) $\bar{V} = \frac{V + V_o}{2} = \frac{10 \text{ m/s} + 59 \text{ m/s}}{2} = \frac{69 \text{ m}}{2 \text{ s}} = 34.5 \text{ m/s}$

STUDENT PROBLEMS

1. A pebble dropped from a bridge hits the water below in 4 seconds. Find (a) the speed (m/s) with which it hits the water and (b) the height (m) of the bridge.

((a) 39.2 m/s; (b) 78.4 m)

2. A stone is projected vertically upwards with a speed of 25 m/s. Find (a) the maximum height reached; (b) the time to reach the topmost point; (c) its speed when it returns back to its projection point and (d) the total time to return to the starting point.

((a) 31.9 m; (b) 2.55 s; (c) 25.0 m/s; (d) 5.10 s)

3. A 1000 kg pile driver hammer is dropped from a height of 3 meters. With what speed does it hit the pile?

(7.67 m/s)

4. A truck traveling at 75 mph passes a stopped police car at the instant that the police car begins to accelerate at 10 ft/s^2. How much time lapses before the car overtakes the truck?

(5.14 s)

SECTION 4 - LINEAR MOMENTUM AND ITS CONSERVATION

The Linear Momentum P of an object of mass m and velocity V is

$$P = mV$$

The units of linear momentum are kg m/s and sl ft/s. Linear momentum is a vector whose direction is that of the velocity. For example, a 5 kg body moving at 4 m/s to the right has a linear momentum of P = 20 kg m/s to the right.

Conservation of Linear Momentum. In any collision process between two or more bodies the total linear momentum of all bodies before the collision is equal to the total linear momentum of all bodies after the collision.

For example, if m_1 (velocity u_1) collides with m_2 (velocity u_2) and the respective velocities after the collision are V_1 and V_2, then

$$m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2 \qquad (1)$$

*Remember that the velocities are <u>vector</u> quantities and thus equation (1) above stands for 3 scalar equations:

$$\begin{aligned} m_1u_{1x} + m_2u_{2x} &= m_1V_{1x} + m_2V_{2x} \\ m_1u_{1y} + m_2u_{2y} &= m_1V_{1y} + m_2V_{2y} \qquad (2) \\ m_1u_{1z} + m_2u_{2z} &= m_1V_{1z} + m_2V_{2z} \end{aligned}$$

where u_{1x}, u_{1y}, u_{1z}, V_{1x}, V_{1y}, V_{1z}, ... refer to the various X-, Y- and Z- components of the velocities u_1, u_2, V_1, V_2, (see Section 5 of this chapter and Chapter 2 of Math Study Guide).

We say that linear momentum is a <u>conserved quantity</u>.

LABORATORY

Using an air track, a ballistic pendulum, a model pile driver or other similar apparatus the student should be able to investigate the phenomenon of collisions, and thus study the conservation of linear momentum.

WORKED EXAMPLES

1. A 10 g bullet hits a 1 kg squirrel sitting on a fence, and as a result of the collision the squirrel and bullet move horizontally at 5 m/s. The bullet is initially moving horizontally with a speed u. Find the initial speed of the bullet.

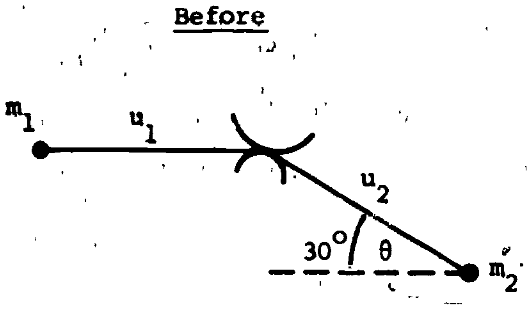
   (Horizontal linear momentum before collision)
   = (Horizontal linear momentum after collision)

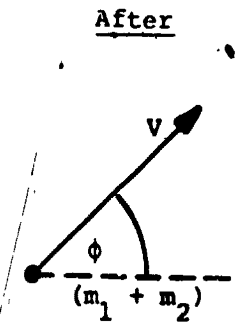
   $$(0.01\ \text{kg})(u) = [(1 + 0.01)\text{kg}][5\ \text{m/s}]$$

   $$u = 505\ \text{m/s}$$

* 2. A 200 lb hockey player is moving due east at 40 ft/s with the puck toward the open net. He is hit by a 220 lb defenseman moving 30° north of west at 22 ft/s, who holds on. What is their speed and direction immediately after collision (both players glide upright on the ice)?

*Optional Material





X-component equation

$$(200\ \text{lb})(40\ \text{ft/s}) + (220\ \text{lb})(-22\ \text{ft/s}\cos 30^\circ)$$

$$= (420\ \text{lb})(V\cos\phi) \qquad (1)$$

Y-component equation

$$(220\ \text{lb})(22\ \text{ft/s}\sin 30^\circ) = (420\ \text{lb})(V\sin\phi) \qquad (2)$$

To find ϕ divide (2) by (1):

$$\tan\phi = \frac{(220)(22)(.500)}{(200)(40) - (220)(22)(.866)}$$

$$= 0.635$$

$$\phi = 32.4^\circ \text{ N of E}$$

substituting in (2)

$$V = \frac{(220)(22)(.500)\ \text{ft/s}}{(420)(.536)} = 10.7\ \text{ft/s}$$

STUDENT PROBLEMS

1. A 10,000 kg truck moving at 4.50 m/s (about 10 mph) strikes a stationary 2,000 kg car in the rear. Find the speed V of the car immediately after the collision. (The cars move together after the collision.)

   (3.75 m/s)

2. A 0.200 kg glider moving at 0.500 m/s on a level air track strikes an initially stationary 0.400 kg glider in such a way that the gliders lock together and move as a unit after the collision. What is the composite glider's speed after the collision?

   (0.167 m/s)

3. A glider at rest at one end of a level air track has a special gun on it which shoots ball bearings in a backwards direction. Initially, the glider, gun and ball bearings have a mass of 400 g and are at rest. If the gun fires three ball bearings successively at 10 m/s relative to the glider, and if each ball bearing has a mass of 50 g, what is the speed of the glider and attached gun immediately after the third ball bearing has been fired?

(6.00 m/s)

SECTION 5 - NEWTON'S LAWS OF MOTION: GRAVITATION

A force may be thought of as a push or pull. Forces are measured in SI units of newtons (N) and English units of pounds (lb). Forces have directions associated with them. Usually these directions are specified by reference to a typical rectangular coordinate system (XY-axes in plane of paper).

Force Components. Any arbitrarily directed force F can be resolved into its components along the 3 coordinate axes. In two dimensions $F_X = F \cos \theta$ and $F_Y = F \sin \theta$, where θ is the angle F makes with the X-axis.

For example a 10 N force making an angle of 30° with the X-axis has X- and Y- components.

$$F_X = F \cos \theta = (10\text{ N})(\cos 30^\circ) = (10\text{ N})(0.866) = 8.66\text{ N}$$
$$F_Y = F \sin \theta = (10\text{ N})(\sin 30^\circ) = (10\text{ N})(0.500) = 5.00\text{ N}$$

Force Combination. The total force acting on an object is equal to the sum of all of the separate forces acting on the object. Proper attention must be paid to the usual sign conventions.

For example, if a vertically upwards force of 10 N and a vertically downwards force of 15 N act on an object, the net force acting is 5 N, vertically downwards.

Newton's Laws of Motion

1. A body left alone (no unbalanced forces acting) maintains a constant velocity.

2. The time rate of change of momentum is equal to the unbalanced force acting,

$$F = \frac{\Delta p}{\Delta t} = \frac{\Delta (mV)}{\Delta t}$$

In most problems mass remains constant and then Newton's second law becomes

$$F = m \frac{\Delta V}{\Delta t} = ma$$

This states that an unbalanced force acting on a body will produce an acceleration which is directly proportional to the force and inversely proportional to the mass.

3. For every <u>action</u> (force) F, there is an equal and opposite <u>reaction</u> (force) -F. (F and -F act on <u>different bodies.</u>)

If mass m is measured in kg and acceleration a is measured in m/s^2, then force F is measured in newtons (N). If F is in pounds and a is in ft/s^2, then m is measured in slugs (sl).

<u>Law of Universal Gravitation</u>

Any two masses m_1 and m_2 whose centers are separated by a distance r attract each other with a force F given by

$$F = G \frac{m_1 m_2}{r^2}$$

when m_1 and m_2 are in kg and r is in meters, then G, a constant of proportionality, is

$$G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$$

LABORATORY

Using such equipment as meter sticks, electric timers and an air track, the student should be able to measure the accelerations of gliders which are descending a sloping air track, or are being subjected to horizontally directed forces on a level track. From such measurements the student should be able to infer the forces acting.

WORKED EXAMPLES

1. Find the force needed to give a 5 kg mass an acceleration of 8 m/s^2.

$$\begin{aligned} F &= ma \\ &= (5 \text{ kg})(8 \text{ m/s}^2) \\ &= 40 \text{ kg m/s}^2 \\ &= 40.0 \text{ N} \end{aligned}$$

2. A 3200 lb automobile is traveling at 60 mph (88 ft/s) on a straight, level road when the driver sees an obstacle ahead. He applies the brakes and comes to a stop in 250 ft. Find the retarding force supplied by the brakes.

$$v^2 = v_o^2 + 2\,as$$

$$a = \frac{v^2 - v_o^2}{2\,s} = \frac{0 - (88\ \text{ft/s})^2}{2(250\ \text{ft})} = \frac{-7744}{500}\ \text{ft/s}^2 = -15.5\ \text{ft/s}^2$$

$$F = ma = \frac{W}{g}\,a = \frac{3200}{32\ \text{ft/s}^2}(-15.5\ \text{ft/s}^2)$$

$$= (100\ \text{sl})(-15.5\ \text{ft/s}^2)$$

$$= -1550\ \text{lb} = -1.55 \times 10^3\ \text{lb}$$

The minus sign shows that the force is acting in a direction opposite to the motion.

3. A 6400 lb elevator is accelerated upwards at 4 ft/s^2. Find the tension T in the elevator cables.

$$F = ma$$

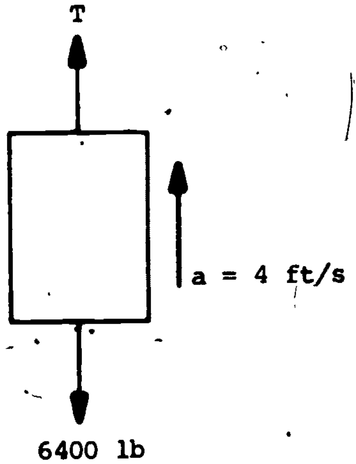
or

$$(T - 6400\ \text{lb}) = \frac{6400\ \text{lb}}{32\ \text{ft/s}^2}(4\ \text{ft/s}^2)$$

$$T = (800 + 6400)\ \text{lb}$$

$$= 7200\ \text{lb}$$

$$= 7.20 \times 10^3\ \text{lb}$$



4. Solve problem 3 above for a downwards acceleration of 4 ft/s^2.

$$F = ma$$

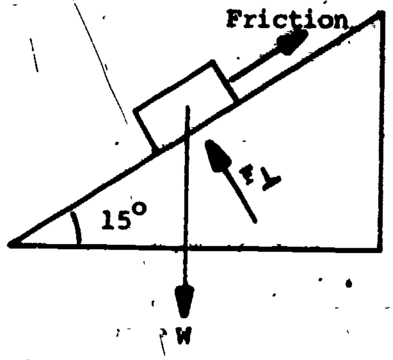
or

$$(T - 6400\ \text{lb}) = (200\ \text{sl})(-4\ \text{ft/s}^2)$$

$$T = (6400 - 800)\ \text{lb}$$

$$= 5600\ \text{lb} = 5.60 \times 10^3\ \text{lb}$$

5. A 1500 kg automobile is parked on a 15° slope. What retarding force must the parking brake supply if the auto is not to roll down the hill?

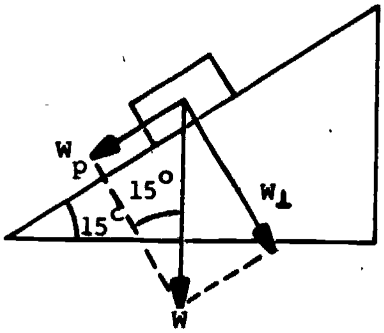


W = weight

$F\perp$ = up push of surface

Friction = frictional force acting

Resolve the weight W into components parallel (W_p) and perpendicular ($W\perp$) to the surface.



$$W_p = W \sin 15^\circ$$
$$= (1500\ kg)(9.8\ m/s^2)(0.262)$$
$$= 3851\ N$$

Friction force must balance W_p

= Friction force = 3851 N,

$= 3.85 \times 10^3 N$

6. Estimate the gravitational force between Raquel Welch and Jim Brown when both are standing facing each other. If these two objects are strongly attracted to each other, is that attraction likely a result just of this force? Assume their centers are 75 cm apart. Let the weights be 54.5 and 90.9 kg.

$$F = G\frac{m_1 m_2}{r^2}$$

$$= \frac{(6.67 \times 10^{-11}\ Nm^2/kg^2)(54.5\ kg)(90.9\ kg)}{(0.75\ m)^2}$$

$$= 5.87 \times 10^{-7}\ N$$

NOT LIKELY!

STUDENT PROBLEMS

1. What upwards force must be exerted on a 15 kg mass in order to cause it to fall downwards with an acceleration of 5 m/s^2?

   (72.0 N)

2. A 3200 lb automobile accelerates from 0 to 60 mph in 12 seconds. Find the force required to produce this acceleration.

   (733 lb)

3. A 1500 kg automobile is moving up a 10^{o} slope at a constant speed of 15 mph. What force must be supplied by the auto in order to maintain this speed? (Neglect frictional effects.)

   (2.57×10^3 N)

4. A 160 lb man is standing on a bathroom scale in an elevator. What will be the reading on the scale if the elevator accelerates upward at 4 ft/s^2? Downwards at 4 ft/s^2?

   (180 lb, 140 lb)

5. If the gravitational force between 2 bodies is found to be F when they are 10 m apart, find the force when they are 20 m apart.

   (F/4)

SECTION 6 - WORK; ENERGY; POWER

Work. When a force F acts on a body and moves it a distance s along the direction of the force, the work U done by the force F is

$$U = Fs$$

Work is measured in joules (J) or foot-pounds (ft-lb). 1 J = 1 Nm.

For example, if an upwards force of 5 N acts on a 10 kg body and moves it 5 m upwards the work done by the 5 N force is (5 N)(5 m) = 25.0 J.

Energy measures the ability of a body to do work. Energy is also measured in joules and foot-pounds.

Potential Energy (E_p) measures a body's ability to do work because of its position. The gravitational potential energy E_p of a body of mass m (weight W) which is a distance h above an arbitrary reference level is

$$E_p = mgh = Wh$$

The body can be thought of as acquiring this potential energy by being lifted (by an external force) through this distance h.

For example, a 5 kg mass which is 10 m above a certain reference level has a potential energy of (5 kg)(9.8 m/s^2)(10 m) = 490 J. In returning to this lower level the body can do 490 J of work.

Kinetic Energy (E_k) measures an object's ability to do work because of its motion. The kinetic energy E_k of a body of mass m and velocity V is

$$E_k = \frac{1}{2} mV^2$$

For example, a 100 kg body moving at a velocity of 30 m/s has a kinetic energy of (0.5)(100 kg)(30 m/s)^2 = 4.50 x 10^4 J. In stopping, this body can do 45,000 J of work (assuming no frictional losses).

Conservation of Energy. Energy can neither be created nor destroyed, but only transformed from one type to another. In motions resulting from the action of the gravitational force

$$E_k + E_p = \text{constant}$$

For example, a stationary 5 kg body 4 meters above a reference level has E_p = 196 J, E_k = 0. As it falls towards the reference level it loses E_p but speeds up and then gains an equal amount of E_k. At all points in the motion, $E_k + E_p$ = 196 J.

Power P measures the rate at which work is done.

$$P = \frac{\text{work}}{\text{time}} = \frac{U}{t} .$$

The units of power are joule/second = watt (W) or ft-lb/s.

Work done = (Power)(time); U = Pt

Some useful conversion factors are:

$$\begin{aligned} 1\text{ W} &= 1\text{ J/s} \\ 1\text{ kilowatt (kW)} &= 1000\text{ W} \\ 1\text{ kilowatt hour (kW-hr)} &= (1\text{ kW})(1\text{ hr}) = 3.60 \times 10^6\text{ J} \\ 1\text{ horsepower (hp)} &= 550\text{ ft-lb/s} = 33{,}000\text{ ft-lb/min} \\ &= 746\text{ W} \end{aligned}$$

For example, if a 2 kg mass is lifted 5 m in 3 s the power exerted is

$$P = \frac{(2\text{ kg})(9.8\text{ m/s}^2)(5\text{ m})}{3\text{ s}} = \frac{98\text{ J}}{3\text{ s}} = 32.7\text{ W}$$

An elastic collision is one in which both linear momentum and kinetic energy are conserved.

A partially elastic (partially inelastic) collision is one in which linear momentum is conserved but some kinetic energy is lost; the objects separate after the collision.

An inelastic collision is one in which linear momentum is conserved and some kinetic energy is lost; the objects stick together after the collision.

LABORATORY

Through the use of such a device as a model pile driver or a ballistic pendulum the student should be able to measure potential energy, kinetic energy and work, and study elastic, partially elastic, and inelastic collision phenomena.

WORKED EXAMPLES

1. Compute the work done by a pump which discharges 800 gallons of water into a tank 100 feet above the intake. The weight density* of water is 62.4 lb/ft^3 and 1 gallon = 0.134 ft^3.

$$\text{work} = mgh = (\text{weight})(\text{height})$$

$$\text{weight} = (800\ \text{gal})\left(\frac{0.134\ \text{ft}^3}{\text{gal}}\right)\left(\frac{62.4\ \text{lb}}{\text{ft}^3}\right)$$

$$= 6689\ \text{lb} = 6.69 \times 10^3\ \text{lb}$$

$$\text{work} = (6689\ \text{lb})(100\ \text{ft})$$

$$= 668{,}900\ \text{ft lb}$$

$$= 6.69 \times 10^5\ \text{ft lb}$$

2. A waterfall is 300 meters high. Find the potential energy of 1 kilogram of water at the top of the falls. What is the velocity of this kilogram of water when it is 150 m above the bottom of the falls. (Assume water is at rest just before it starts to fall.)

$$\text{At top, } E_p = mgh = (1\ \text{kg})(9.8\ \text{m/s}^2)(300\ \text{m}) = 2940\ \text{J} = 2.94 \times 10^3\ \text{J}$$

$$E_k = 0$$

*See Chapter V, Section 1, for a discussion of density.

Halfway down, $E_p = \frac{2940}{2}$ J = 1470 J = 1.47×10^3 J

$$E_k = 1470 \text{ J} = \frac{1}{2} mV^2 = \frac{1}{2}(1 \text{ kg})(V^2)$$

$$V^2 = 2940 \text{ m}^2/\text{s}^2$$

$$V = 54.2 \text{ m/s}$$

3. In a model pile driver a 4 kg mass falls 1 m and drives a nail 3 mm into a wood block. Find the original potential energy of the 4 kg mass, its kinetic energy and velocity right before impact with the nail, and the average force it exerts on the nail.

At beginning, $E_p = mgh = (4 \text{ kg})(9.8 \text{ m/s}^2)(1 \text{ m}) = 39.2$ J

$$E_k = 0$$

Just before impact, $E_k = \frac{1}{2} mV^2 = 39.2 \text{ J} = \frac{1}{2}(4 \text{ kg})(V^2)$

$$E_p = 0$$

$$V^2 = 19.6 \text{ m}^2/\text{s}^2$$

$$V = 4.43 \text{ m/s}$$

Work done = 39.2 J = $(\overline{F})(d) = (\overline{F})(0.003 \text{ m})$

$$\overline{F} = \frac{39.2 \text{ J}}{0.003 \text{ m}} = 13{,}067\text{N} = 1.31 \times 10^4 \text{ N}$$

4. An average student weighs 800 N. (a) Compute the increase in potential energy of this student if he walks up a flight of stairs to a vertical height 4 m above his initial position. (b) If he walks up a spiral staircase instead of an ordinary stairway, will the answer to part (a) change? Why or why not? (c) If he runs instead of walking, will your answer be different? (d) If he climbs the 4 m in 4 s, what is his power output (in horsepower)?

(a) Change in E_p = mgh = Wh = (800 N)(4 m) = 3200 J = 3.20×10^3 J

(b) Same answer as (a), only the <u>vertical</u> height counts.

(c) Same as answer (a). Speed does not enter into E_p idea. How he gets to the top has no effect on final E_p.

(d) Power = $\frac{\text{work}}{\text{time}} = \frac{3200 \text{ J}}{4 \text{ s}} = 800 \frac{\text{J}}{\text{s}} = (800 \text{ W})(\frac{1 \text{ hp}}{746 \text{ W}}) = 1.07$ hp

<u>Is this realistic?</u>

5. For worked example #1 of Section 4, find the total kinetic energy both before and after the collision. Consider only the horizontal part of the motion.

<u>Before collision:</u> $E_k = \frac{1}{2}(0.01\ \text{kg})(505\ \text{m/s})^2$

$$= 1275\ \text{J} = 1.28 \times 10^3\ \text{J}$$

<u>After collision:</u> $E_k = \frac{1}{2}(1.01\ \text{kg})(5\ \text{m/s})^2$

$$= 12.6\ \text{J}$$

Percentage of original kinetic energy which is lost

$$= \frac{(1275 - 12.63)}{1275} \times 100\% = 99.0\%$$

What happens to the kinetic energy that disappeared?

6. Two like billiard balls (200 g each) collide head on. One ball is at rest before the collision and the other one is moving at 5 m/s. Assume elastic collision and motion along a straight line. Find the velocities of the balls after the collision.

$$m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2$$

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2$$

$$(0.2\ \text{kg})(5\ \text{m/s}) + (0) = (0.2\ \text{kg})(V_1) + (0.2\ \text{kg})(V_2)$$

$$\frac{1}{2}(0.2\ \text{kg})(5\ \text{m/s})^2 + 0 = \frac{1}{2}(0.2\ \text{kg})V_1^2 + \frac{1}{2}(0.2\ \text{kg})V_2^2$$

or

$$5\ \text{m/s} = V_1 + V_2 \qquad (1)$$

$$25\ \text{m}^2/\text{s}^2 = V_1^2 + V_2^2 \qquad (2)$$

from (1)

$$V_1 = 5\ \text{m/s} - V_2$$

put this in (2)

$$25\ \text{m}^2/\text{s}^2 = 25\ \text{m}^2/\text{s}^2 = (5\ \text{m/s} - V_2)^2 + V_2^2$$

$$2V_2^2 = (10\ \text{m/s})V_2$$

$$V_2 = 5.00 \text{ m/s}$$

and from (1)

$$V_1 = 0$$

Conclusion: As a result of the collision the objects merely exchange velocities.

STUDENT PROBLEMS

1. A body of mass 4 kg is raised a distance of 10 m in 15 seconds. Find the work done and the power exerted.

   (392 J, 26.1 W)

2. A 10 kg object is thrown vertically downwards from a bridge at 15 m/s. Find its kinetic energy after 5 seconds.

   (2.05×10^4 J)

3. Calculate the horsepower needed to lift a 100 pound mass to a height of 30 feet in one minute.

   (0.0909 hp)

4. Find the potential energy gain as a 5 kg mass is raised a distance of 10 m.

   (490 J)

5. A 3200 pound automobile is traveling at a speed of 30 mph down a 30° slope. What force must the brakes exert if the auto is to be brought to a stop in 250 feet? (Assume auto is still on incline.)

   (1.99×10^3 lb)

END OF CHAPTER PROBLEMS

1. A ball is dropped from a height of 2 meters above a tile floor and rebounds to a height of 1 meter. Find the velocity of the ball immediately after its collision with the floor. What percentage of its original kinetic energy is lost during the collision? What happens to this kinetic energy?

   (4.43 m/s; 50.0%)

2. A 2000 kg automobile is pulling a trailer on a level road at a steady speed of 20 m/s. The force of rolling friction on the trailer is 900 newtons; this force opposes the motion. (a) What is the direction and magnitude of the force which the trailer exerts on the car? (b) What is the direction and magnitude of the force which the road exerts on the car?

   ((a) 900 N, backwards, (b) 900 N, forward, 1.96×10^4 N, up)

3. A person leaning over the side of a bridge throws a stone upwards with a speed of 30 m/s. The bridge is 100 m above the water. Find the maximum height achieved by the stone and the speed with which it hits the water. Find the total time of flight.

(45.9 m; 53.5 m/s; 8.52 s)

4. An inclined plane makes an angle of 30° with the horizontal. Find the force needed to move a 50 N box (a) up the plane with an acceleration of 5 m/s^2 and (b) down the plane with an acceleration of 5 m/s^2. Neglect friction and apply all forces parallel to the incline.

((a) 50.5 N, up plane; (b) 0.510 N, down plane)

5. A 50 N block is projected up an inclined plane which makes an angle of 30° with the horizontal at a speed of 15 m/s. How far up the plane will it go before coming to a stop? How long will it take to return to the starting point? Neglect all frictional effects.

(23.0 m; 6.12 s)

CHAPTER III

ROTATIONAL MOTION

SECTION 1 - CENTRIPETAL FORCE

A body moving with constant speed in a circle is said to have uniform circular motion. This is an accelerated motion since the direction of the velocity is continually changing.

A body executing uniform circular motion (speed V) has an acceleration which is directed perpendicular to the velocity V and, hence, points toward the center of the circle. The value of this centripetal (center-seeking) acceleration a is

$$a = \frac{V^2}{r}$$

where r is the radius of the circle.

For example, for a 5 kg mass moving 10 m/s around a circle of 4 m radius the centripetal acceleration is

$$a = \frac{V^2}{r} = \frac{(10\ m/s)^2}{4\ m} = \frac{100\ m^2/s^2}{4\ m} = 25.0\ m/s^2$$

A body executing uniform circular motion must have an unbalanced force F acting on it. The force is directed towards the center of the circle. This centripetal force F has a magnitude of

$$F = \frac{mV^2}{r}$$

For example, the 5 kg mass above, which moves at a speed of 10 m/s in a circle of radius 4 m, has acting on it a centrally directed force of

$$\frac{(5\ kg)(10\ m/s)^2}{4\ m} = 125\ N\ .$$

LABORATORY

Using such devices as commercially available centripetal force apparatus or air tables the student should be able to investigate centripetal forces and accelerations.

WORKED EXAMPLES

1. A body of mass 0.200 kg is rotating in a horizontal circle of radius 50 cm at 120 revolutions per minute (rpm). (a) Calculate the centripetal force acting and (b) find the period T of this motion (the period is the time for one complete revolution).

   (a) One revolution in this problem means a distance of $2\pi r = (2\pi)(0.500\text{ m}) = 3.14$ meters is traveled.

   $$120\,\frac{\text{rev}}{\text{min}} = (120\,\frac{\text{rev}}{\text{min}})(\frac{3.1416\text{ m}}{\text{rev}})(\frac{1\text{ min}}{60\text{ s}}) = 6.28\text{ m/s}$$

   $$F = \frac{mV^2}{r} = \frac{(0.200\text{ kg})(6.28\text{ m/s})^2}{0.500\text{ m}} = 15.8\text{ N}$$

   (b) $$\text{Period} = T = \frac{\text{Circumference}}{\text{Speed}} = \frac{3.14\text{ m}}{6.28\text{ m/s}} = 0.500\text{ s}$$

2. Why is it harder for a car to make a given turn at high speed than at low speed?

   For a turn of a given radius the centripetal force needed to produce the circular motion of a given car depends only upon the square of the speed of the car; the higher the speed, the more centripetal force needed.

   The friction between the road and the tires is the mechanism that produces the needed centripetal force. At higher speeds the needed frictional force just may not be available and, hence, the car will skid.

3. The "normal" human body can safely stand an acceleration of approximately 9 times the acceleration of gravity. With what minimum radius of curvature may a pilot safely pull out of a dive in which the plane is traveling at 135 m/s (300 mph)?

   At the bottom of the dive two forces act on the pilot: (1) the downwards force mg of gravity and (2) the upwards force F_{up} due to the force of the seat on him. Together these two forces must add up to the needed upward centripetal force.

   $$F_{up} - mg = \frac{mV^2}{r}$$

   $$F_{up} = mg + \frac{mV^2}{r}$$

   If he were sitting on a scale on his seat it would also read F_{up}. Why?

Here,

$$F_{up} = 9\,mg = mg + \frac{mV^2}{r}$$

$$8\,g = \frac{V^2}{r}$$

$$r = \frac{V^2}{8\,g} = \frac{(135\ m/s)^2}{(8)(9.8\ m/s^2)} = \frac{18225}{78.4}\ m$$

$$= 232\ m$$

STUDENT PROBLEMS

1. The precision clock in a physics laboratory has a second hand 10.7 cm long, which makes one revolution per second. (a) Find the speed of the tip of this hand. (b) Find the centripetal acceleration of this tip.

   ((a) 0.672 m/s; 4.22 m/s^2)

2. A man is standing at the equator. Taking the radius of the earth to be 6.40×10^6 m, what is the speed of the man with respect to the center of the earth. Find his centripetal acceleration. Compare your result with g.

   (465 m/s; 0.0338 m/s^2; 0.345%)

3. The gravitational force of the sun on the earth is 3.08×10^{22} N.

   (a) Find the centripetal acceleration of the earth in its orbit (radius of 1.496×10^8 km, mass of 6.00×10^{24} kg).

   (b) Find the speed of the earth in its orbit using result of part (a).

   (c) Find the period of the earth's revolution about the sun in seconds and in days, using the result of part (b). Compare with your calendar.

   (a) 0.00513 m/s^2; (b) 2.77×10^4 m/s;

   (c) 3.39×10^7 s, 393 days

SECTION 2 - ROTATIONAL ANGLE, VELOCITY, ACCELERATION

Angular displacement (rotational angle) specifies the angle through which a body has rotated. It is normally measured in radians and is denoted by θ. One radian is the angle subtended at the center of a circle by an arc equal in length to the radius.

$$1\ \text{radian} = 1\ \text{rad} = 57.3^\circ$$

$$2\pi\ \text{rad} = 360^\circ = 1\ \text{revolution (rev)}$$

Radian is a pure (dimensionless) number.

For example, if a grinding wheel rotates 2 complete times we say that it has experienced an angular displacement of 4π rad (or 720° or 2 rev).

Angular displacement has direction: counterclockwise = positive and clockwise = negative.

Angular velocity specifies the rate at which a body is turning through an angle. It is normally measured in rad/s and is denoted by the Greek letter omega - ω.

If a body turns through an angle θ in a time t, its average angular velocity $\bar{\omega}$ is given by

$$\bar{\omega} = \frac{\theta}{t} = \frac{\text{Angle Turned Through}}{\text{Time}}$$

For example, a body rotating at a rate of 3 revolutions per minute has an angular velocity of 6π rad/60·sec = $\pi/10$ rad/s.

Angular velocity has a direction: counterclockwise = positive and clockwise = negative.

Angular Acceleration specifies the rate at which angular velocity changes. It is measured in rad/s^2 and is denoted by the Greek letter alpha - α.

If the angular velocity of a body changes uniformly from ω_o to ω in a time t, then

$$\alpha = \frac{\omega - \omega_o}{t} = \frac{\text{Change in Angular Velocity}}{\text{Time}}$$

An increasing (in the counterclockwise sense) angular velocity implies a positive angular acceleration; a decreasing angular velocity implies a negative angular acceleration.

For example, if a grinding wheel changes its angular velocity from +50 rad/s to +100 rad/s in 10 s, its angular acceleration is

$$\alpha = \frac{(100-50)\text{ rad/s}}{10\text{ s}} = 5.00\text{ rad/s}^2$$

The linear quantities of arc length s, (radius r) velocity V and acceleration a are related to the corresponding angular quantities of angular displacement θ, angular velocity ω and angular acceleration α such that

$$s = r\theta$$
$$V = r\omega$$
$$a = r\alpha$$

Corresponding analogies exist between the equations for uniformly accelerated motion and the equations for uniform angular acceleration,

$$\theta = \bar{\omega}t = \frac{\omega + \omega_o}{2} t$$

$$\theta = \omega_o t + (1/2)\alpha t^2$$

$$\omega = \omega_o + \alpha t$$

$$\omega^2 = \omega_o^2 + 2\alpha\theta$$

LABORATORY

Using such equipment as a regularly rotating body (e.g., an electric fan), various time measuring devices such as stroboscopes and electric timers and various length measuring devices such as meter sticks, vernier and micrometer calipers, the student should be able to measure the various angular quantities (displacement, velocity, acceleration) and relate them to each other and to the corresponding linear quantities.

SOLVED PROBLEMS

1. A 0.500 meter radius grinding wheel is rotating at 180 rpm. Find (a) the angular velocity of this wheel and (b) the linear speed of a point on its periphery.

(a) $\omega = (180 \frac{\text{rev}}{\text{min}})(\frac{1 \text{ min}}{60 \text{ s}})(\frac{2\pi \text{ rad}}{\text{rev}}) = 6\pi \text{ rad/s} = 18.8 \text{ rad/s}$

(b) $V = \omega r = (18.8 \frac{\text{rad}}{\text{s}})(0.500 \text{ m}) = 9.40 \text{ m/s}$

2. An electric motor revolving at 3600 rpm is turned off and slows down uniformly to a stop in 10 seconds. (a) Find the angular acceleration of the motor shaft and (b) find the total angular displacement as it slows to a stop.

(a) $3600 \frac{\text{rev}}{\text{min}} = (3600 \frac{\text{rev}}{\text{min}})(\frac{2\pi \text{ rad}}{1 \text{ rev}})(\frac{1 \text{ min}}{60 \text{ s}}) = 120\pi \frac{\text{rad}}{\text{s}}$

$$\omega = \omega_o + \alpha t \text{ or } \alpha = \frac{\omega - \omega_o}{t} = \frac{0 \text{ rad/s} - 120\pi \text{ rad/s}}{10 \text{ s}} = -12\pi \text{ rad/s}^2$$

(b) $\theta = \omega_o t + (1/2)\alpha t^2 = (120\pi \frac{\text{rad}}{\text{s}})(10 \text{ s}) + (1/2)(-12\pi \frac{\text{rad}}{\text{s}^2})(10 \text{ s})^2$

$= 1200\pi \text{ rad} - 600\pi \text{ rad} = 600\pi \text{ rad} = 1.88 \times 10^3 \text{ rad} = 300 \text{ rev}$

3. A 3 speed electric fan is revolving at 1200 rpm (high speed). It is then switched to low speed (600 rpm) and it is noted on a revolution counter that the fan makes 450 revolutions in going from high speed to low speed. Find (a) the angular acceleration, (b) the time required to go from high speed to low speed.

(a) $1200 \text{ rpm} = (1200 \frac{\text{rev}}{\text{min}})(\frac{1 \text{ min}}{60 \text{ s}})(\frac{2\pi \text{ rad}}{1 \text{ rev}}) = 40\pi \text{ rad/s}$

Similarly, $600 \text{ rpm} = 20\pi \text{ rad/s}$; $450 \text{ rev} = 900\pi \text{ rad}$

$$\omega^2 = \omega_o^2 + 2\alpha\theta$$

$$\alpha = \frac{\omega^2 - \omega_o^2}{2\theta} = \frac{(20\pi \text{ rad/s})^2 - (40\pi \text{ rad/s})^2}{(2)(900\pi \text{ rad})}$$

$$= \frac{(400 - 1600)\pi^2}{1800\pi} \frac{\text{rad}}{\text{s}^2} = \frac{-2\pi}{3} \frac{\text{rad}}{\text{s}^2}$$

(b) $\theta = \omega_o + (1/2)\alpha t^2 = \frac{\omega_o + \omega}{2} t$

$$t = \frac{2\theta}{\omega + \omega_o} = \frac{(2)(900\pi \text{ rad})}{(20\pi + 40\pi) \text{ rad/s}} = \frac{1800\pi \text{s}}{60\pi} = 30.0 \text{ s}$$

STUDENT PROBLEMS

1. An electric fan goes from 600 rpm to 900 rpm in 20 s. Find (a) its angular acceleration and (b) the angle turned through during this accelerating process.

((a) $1.57 \frac{\text{rad}}{\text{s}^2}$; (b) 1.57×10^3 rad)

2. An electric drill is rotating at 3000 rpm as it is drilling a hole in a piece of wood. It then encounters a knot in the wood and stalls (stops rotating) almost immediately (it makes 50 revolutions in going from 3000 rpm to a dead stop). (a) Find the angular acceleration and (b) find the time involved in the stalling process.

((a) -157 rad/s^2; (b) 2.00 s)

3. The beaters on an electric mixer are rotating at 600 rpm. The prongs of the beater have a radius of about 3 inches (7.50 cm). (a) Find the angular velocity of the beaters and (b) find the linear velocity of one of the prongs.

((a) 62.8 rad/s; (b) 4.71 m/s (10.5 mph))

SECTION 3 - TORQUE AND STATIC EQUILIBRIUM

Torque refers to the effectiveness of a force in producing rotation about an axis. It is defined as the product of the force F acting and the perpendicular distance ℓ from the axis of rotation to the line of action of the force. ℓ is referred to as the lever arm.

$$\text{Torque} = \tau = (F)(\ell)$$

Torque is usually measured in newton-meters (Nm) or pound-feet (lb ft). Torques are directional and are usually referred to as counterclockwise (positive) or clockwise (negative).

Torque Combination. The total torque acting on an object is equal to the sum of the separate torques acting, making proper allowance for the sign convention defined above. For example, if a clockwise torque of 50 Nm and a counterclockwise torque of 150 Nm act on a body, the net torque acting is 100 Nm, counterclockwise.

The center of mass of a rigid body is a point at which we can consider all of the mass of the body to be concentrated. For many calculations of interest a rigid body may be replaced by a point mass (mass equal to the actual mass of the body) located at the center of mass position of the rigid body. In a uniform gravitational field the terms center of mass and center of gravity are synonymous.

For example, the center of mass of a uniform meter stick would be located at its 50 cm mark while the center of mass of a doughnut would be located at the center of the hole.

An object is said to be in static equilibrium if the total force acting on the object is zero and the total torque acting on the object is zero. That is,

$$\sum F_X = 0$$

$$\sum F_Y = 0$$

$$\sum F_Z = 0$$

$$\sum \tau = 0$$

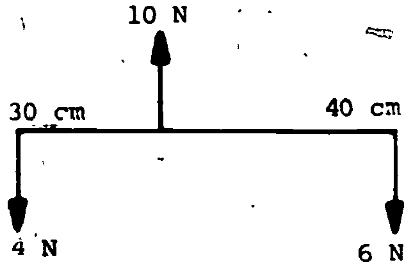
These are algebraic sums.

LABORATORY

Using a balance (or other similar devices) the students should be able to develop the laws of static equilibrium. Also, they should be able to determine unknown masses using such a balance - both for cases where the balance is supported at its center of gravity and for cases where the point of support is not at the center of gravity.

WORKED EXAMPLES

1. A rigid stick is acted upon by the indicated forces. Is this stick in static equilibrium? Neglect the weight of the stick.



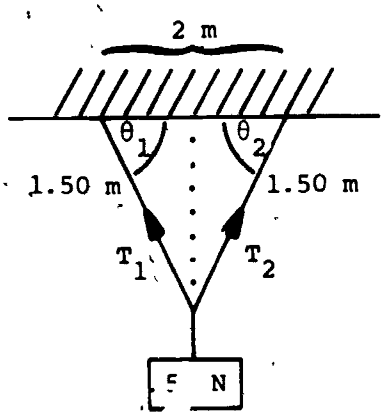
The total force acting on the stick is zero: 10 N, up and (6 + 4) N, down.

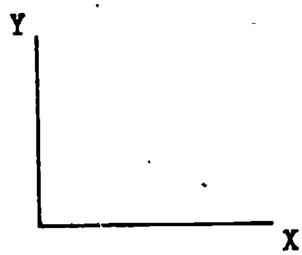
For the torque calculation choose the axis to be at the point of application of the 10 N force.

$$\sum \tau = (4\ \text{N})(0.400\ \text{m}) - (6\ \text{N})(0.300\ \text{m}) = (1.60 - 1.80)\text{Nm} = -0.200\ \text{Nm}$$

Therefore, clockwise rotation will occur.

2. A 50 N board is supported as shown. Find the tension in each section of the rope.





$$\sum F_X = 0 = T_2 \cos\theta - T_1 \cos\theta$$

Therefore $T_2 = T_1 = T$

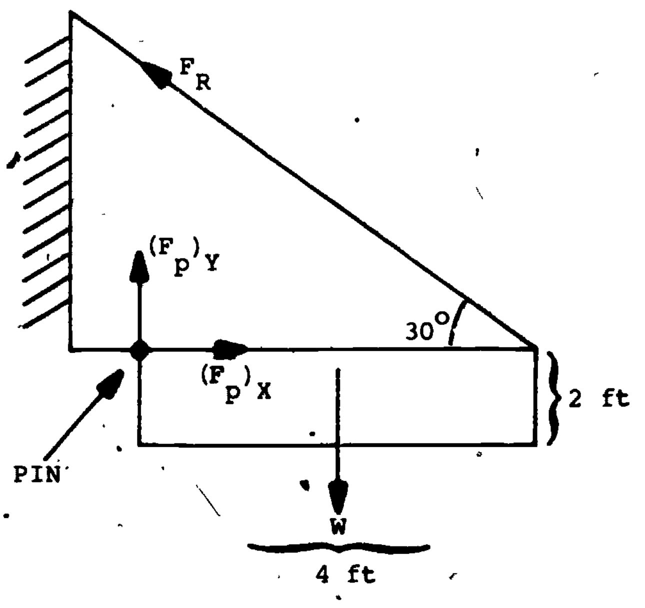
$$\sum F_Y = 0 = 2T \cos\theta - 50 \text{ N} \qquad (1)$$

Now $\cos\theta = \dfrac{1 \text{ m}}{1.50 \text{ m}} = 0.667$

Therefore, from (1)

$$T = \frac{50 \text{ N}}{2 \cos\theta} = \frac{50 \text{ N}}{(2)(0.667)} = \frac{50 \text{ N}}{1.334} = 37.5 \text{ N}$$

3. A 500 lb sign is supported as shown below. The sign is uniform in density. Find the force exerted by the rope and the force exerted by the pin.



$(F_R)_X = F_R \cos 30^o = 0.866\, F_R$ to the left

$(F_R)_Y = F_R \sin 30^o = 0.500\, F_R$, up

Let $(F_p)_X$ = X- part of pin force

$(F_p)_Y$ = Y- part of pin force

$\sum F_X = 0 = -(F_R)_X + (F_p)_X$

or $0.866\, F_R = (F_p)_X$ (1)

$\sum F_Y = 0 = (F_R)_Y + (F_p)_Y - W = 0$

or $0.500\, F_R + (F_p)_Y = 500$ lb (2)

But equations (1) and (2) contain 3 unknowns (F_R), $(F_p)_X$, $(F_p)_Y$ and are hence unsolvable. Another equation is needed.

Let the pin be the point about which torques are taken.

$$\sum \tau = 0$$

or, $-(500 \text{ lb})(2 \text{ ft}) + (F_R)_Y(4 \text{ ft}) = 0$ (3)

(<u>NOTE</u>: $((F_R)_X$, $(F_P)_X$ and $(F_P)_Y$ have lever arms of zero.)

or

$1000 \text{ lb ft} = (0.500)F_R(4 \text{ ft}) = (2.00 \text{ ft})(F$

$F_R = \dfrac{1000 \text{ lb ft}}{2.00 \text{ ft}}$ = force exerted by rope = 500 lb

From (1), $(F_P)_X = 0.866\ F_R = (0.866)(500 \text{ lb}) = 433 \text{ lb}$

and from (2) $(F_P)_Y = 500 \text{ lb} - 0.500\ F_R$

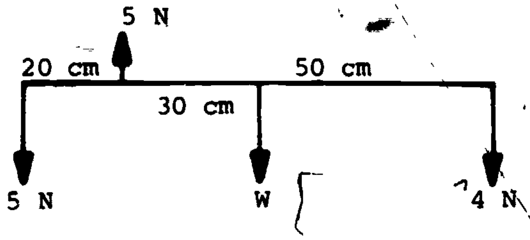
$= 500 \text{ lb} - (0.500)(500 \text{ lb})$

$= 500 \text{ lb} - 250 \text{ lb}$

$= 250 \text{ lb}$

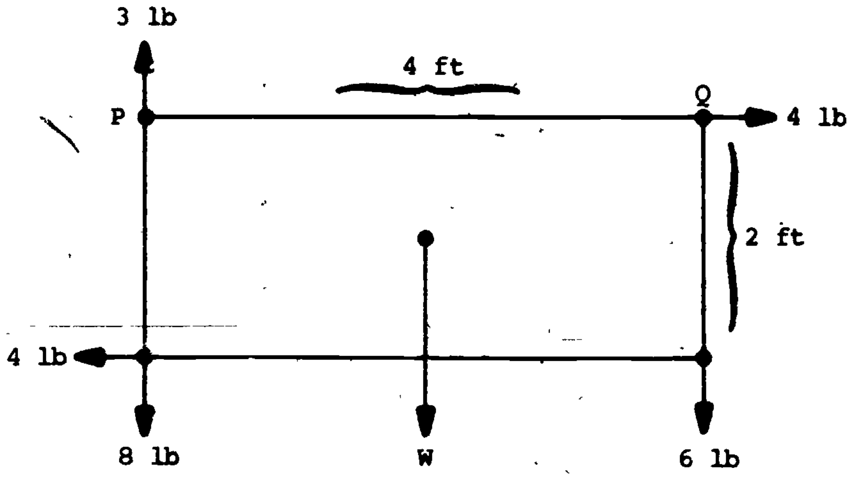
STUDENT PROBLEMS

1. A 200 g meter stick has forces acting on it as shown below. What single vertically directed force (magnitude, direction, and point of application) must be applied to produce static equilibrium?



(5.96 N, up; 66.8 cm from left end)

2. The 5 lb rigid body shown below is acted upon by the indicated forces. What total force acts on the body? Find the torque tending to rotate the object about point P. What upward force applied at point Q would hold the body in static equilibrium?



(16 lb, down; 42.0 ft-lb, clockwise; 10.5 lb)

SECTION 4 - MOMENT OF INERTIA; ROTATIONAL ENERGY

Moment of Inertia measures the resistance a body offers to a change in its angular velocity. It is the angular counterpart of mass. It depends upon both the mass of a body and the distribution of this mass about the axis of rotation of the body. For example, for a point mass m rotating at a perpendicular distance r from a fixed axis, the moment of inertia I is defined to be

$$I = mr^2$$

Any extended body can be thought of as a collection of masses m_1, m_2, m_3, ... located at distances r_1, r_2, r_3, ... from the axis of rotation. Consequently,

$$I = \sum_i m_i r_i^{\,2}$$

$$= m_1 r_1^{\,2} + m_2 r_2^{\,2} + m_3 r_3^{\,2} + \ldots$$

For certain specifically shaped bodies this sum is relatively easy to evaluate using the techniques of the calculus. Some specific results are:

Sphere (axis through center; mass m; radius r; solid) $I = (2/5)mr^2$

Point Mass (mass m, distance r) $I = mr^2$

Thin Ring / Hollow Cylinder / Hoop (mass m, radius r; about its own axis) $I = mr^2$

Thin Uniform Rod (mass m, length ℓ; about an axis perpendicular to rod at its center) $I = (1/12)m\ell^2$

Solid Cylinder; Disk (mass m, radius r, axis through center and perpendicular to its plane face) $I = (1/2)mr^2$

Moment of inertia is measured in units of $kg\text{-}m^2$ or $sl\text{-}ft^2$. For example, a 5 kg mass rotating in a circle of radius 5 m has a moment of inertia I of $I = mr^2 = (5 \text{ kg})(5 \text{ m})^2 = 125 \text{ kg-m}^2$.

The angular acceleration α, torque τ and moment of inertia I are related by the equation

$$\tau = I\alpha$$

Sign conventions should be observed as noted in Section 3. The above equation is the angular counterpart of the linear form of Newton's Second Law, $F = ma$.

For example, an unbalanced torque of 50 Nm acting on a body whose moment of inertia is 100 $kg\text{-}m^2$ will produce an angular acceleration of

$$\alpha = \frac{\tau}{I} = \frac{50 \text{ Nm}}{100 \text{ kgm}^2} = \frac{(50 \text{ kgm/s}^2)(\text{m})}{100 \text{ kgm}^2}$$

$$= 0.500 \text{ rad/s}^2$$

Rotational Energy $(E_K)_R$ is possessed by any rotating body.

$$(E_K)_R = (1/2)I\omega^2$$

$(E_K)_R$ is measured in joules (J) or foot-pounds (ft-lb).

For example, a body with a moment of inertia of 125 $kg\text{-}m^2$ rotating with an angular velocity of 10 rad/s has a rotational kinetic energy of $(1/2)(125 \text{ kg-m}^2)(10 \text{ rad/s})^2 = 6.25 \times 10^3$ J.

If a body is moving as a whole (translating) and rotating simultaneously, its total kinetic energy (E_K) is made up of two parts: $(E_K)_T$ due to its translational motion and $(E_K)_R$ due to its rotational kinetic energy.

$$(E_K) = (E_K)_T + (E_K)_R$$

$$= (1/2)mV^2 + (1/2)I\omega^2$$

Here V represents the velocity of the center of mass of the body and the axis of rotation must be through the center of mass of the body.

An example of this type of motion would be a blackboard eraser which is thrown up into the air in such a way that it rotates as it moves along.

Work U must be done on a rotating object in order to produce an angular acceleration α.

$$U = \tau\theta$$

Work U is measured in joules, torque τ is measured in newton-meters and angle θ is measured in radians.

For example, if a torque of 15 Nm is applied to a grinding wheel as it rotates through 15 rad the work U done, $U = \tau\theta = (15\text{ Nm})(15\text{ rad}) = 225\text{ J}$.

Power P is analogously defined by the equation

$$P = \tau\omega$$

For example, referring to the above data ($\tau = 15$ Nm, $\theta = 15$ rad) and assuming a time of 10 seconds (and uniform angular acceleration) the power delivered to the wheel is $P = \tau\omega = (15\text{ Nm})(15\text{ rad}/10\text{ s}) = 22.5\text{ W}$.

LABORATORY

Using commercially available equipment such as rotatable disks, rings and cylinders as well as a modified electric fan, the student should be able to measure angular displacement, angular velocity, angular acceleration; he should also be able to measure the moment of inertia of the apparatus. Finally, he should be able to relate torque, work and power to the various angular quantities noted above.

SOLVED PROBLEMS

1. An electric fan blade assembly has a mass of 10 kg and an effective moment of inertia of 8 kgm^2. It is rotating at 1200 rpm. Find its rotational kinetic energy.

$$(E_K)_R = (1/2)I\omega^2 = (1/2)(8\text{ kgm}^2)\left[(1200\ \frac{\text{rev}}{\text{min}})(\frac{1\text{ min}}{60\text{ s}})(\frac{2\pi\text{ rad}}{\text{rev}})\right]^2$$

$$= (4)(1600\pi^2)\text{ J} = 6400\pi^2\text{ J} = 6.32 \times 10^4\text{ J}$$

2. The electric fan noted above is operated by a 500 W motor. The shaft which is attached to the motor and fan blade assembly has a diameter of 6 cm. (a) What is the maximum torque available from this motor at this rotational rate? (b) Such an applied torque should produce an acceleration but it obviously doesn't because of the uniform rotation rate of the fan. Explain.

(a) $P = \tau\omega$

$$\tau = \frac{P}{\omega} = \frac{500\text{ W}}{40\pi\text{ rad/s}} = 3.98\text{ Nm}$$

(b) The motor torque of 3.98 Nm is balanced by frictional torques due to bearing and shaft friction. Also there is air friction and energy delivered from the fan in the way of a moving current of air coming out of the fan.

3. A meter stick has a mass of 150 grams. Find its moment of inertia through a transverse axis through its center.

$$I = (1/12)m\ell^2 = (1/12)(0.150\text{ kg})(1\text{ m})^2 = 0.0125\text{ kgm}^2$$

4. A 1 kg ball rolls from rest down a smooth incline 10 meters high. Find its linear speed when it is at the bottom of the incline.

$$(E_K) = (E_K)_T + (E_K)_R = \text{total mechanical energy at bottom}$$

$$= (1/2)mV^2 + (1/2)I\omega^2$$

$$= (1/2)(m)(V^2) + (1/2)(2/5\ mr^2)(V/r)^2$$

$$= (1/2)mV^2 + (1/5)mV^2 = (7/10)mV^2$$

= total mechanical energy at top (conservation of energy)

$$= mgh$$

$$\therefore mgh = (7/10)mV^2$$

$$V^2 = (10/7)gh$$

$$V = \sqrt{(10/7)gh}$$

$$= \sqrt{\frac{(10)(9.8\text{ m/s}^2)(10\text{ m})}{7}} = \sqrt{980/7}\text{ m/s} = \sqrt{140}\text{ m/s}$$

$$V = 11.8\text{ m/s}$$

STUDENT PROBLEMS

1. Compute the moment of inertia of a 5 kilogram wheel having a radius of 0.5 meters. Assume all of the mass of the wheel is in its rim.

(1.25 kgm^2)

2. An electric motor runs at 1800 rpm and is rated as a 200 W motor. How much torque does it deliver?

(1.06 Nm)

3. A cord 4 m long is wrapped around the axle of a wheel. The cord is pulled with a constant force of 30 N. When the cord leaves the axle, the wheel is rotating at 120 rpm. Find the moment of inertia of the wheel and axle.

($1.52\ kgm^2$)

END OF CHAPTER PROBLEMS

1. An electric motor runs at 1800 rpm and is rated at 1200 W. How much torque can it deliver?

(6.37 Nm)

2. The drive wheel of a belt drive attached to an electric motor has a radius of 10 cm. The drive wheel rotates at 1800 rpm. The tension in the belt is 100 N on the slack side and 400 N on the taut side. Find the power transmitted by the belt.

(5.65×10^3 W)

3. A circular disk has a radius of 10 cm and a mass of 2 kg. Find its moment of inertia.

($0.010\ kgm^2$)

4. A dumbbell rotates about an axis through its center of mass, perpendicular to the dumbbell axis. Each mass is 0.50 kg, the distance between the masses is 0.30 m and the angular velocity is 5 rad/s. (a) Find the speed of each mass. (b) Calculate the kinetic energy of the system.

((a) 0.75 m/s; (b) 0.28 J)

5. The flywheel in a typical automobile engine has a mass of 10 kg and a radius of about 20 cm. Assume that the mass is distributed uniformly throughout the disk. (a) Find the rotational kinetic energy of this flywheel at 5000 rpm. (b) Find the kinetic energy of a car moving at 50 mph (assume a mass of 1500 kg). (c) Compare and contrast results of (a) and (b).

((a) 2.74×10^4 J; (b) 3.75×10^5 J;
(c) E_K of flywheel ≈ 10% E_K of auto)

CHAPTER IV

TEMPERATURE AND HEAT

SECTION 1 - TEMPERATURE SCALES AND MEASUREMENTS

There are three temperature scales that are generally used: the Celsius (centigrade) scale, the Fahrenheit scale and the Kelvin (absolute) scale.

On the Celsius scale (^{o}C) water freezes at 0^{o}C and boils at 100^{o}C; on the Fahrenheit scale (^{o}F) water freezes at 32^{o}F and boils at 212^{o}F. On the Kelvin scale (^{o}K) water freezes at 273^{o}K and boils at 373^{o}K. *

The conversion factors relating the various scales are

$$\text{Temp. } (^{o}C) = 5/9\ [\text{Temp } (^{o}F) - 32]\ ^{o}C$$

$$\text{Temp. } (^{o}F) = (9/5[\text{Temp } (^{o}C) + 32])^{o}F$$

$$\text{Temp. } (^{o}K) = [\text{Temp } (^{o}C) + 273]^{o}K$$

Also, a change of 1 centigrade degree (C^{o}) is equivalent to a change of 9/5 Fahrenheit degree (F^{o}) and to a change of 1 Kelvin degree (K^{o}).

For example, a temperature of 100^{o}F is equivalent to

$$100^{o}\ F = 5/9\ [100 - 32]\ ^{o}C = 37.8^{o}C$$

$$= [37.8 + 273]^{o}K = 311^{o}K$$

When a solid object of length ℓ is subjected to a temperature change ΔT its length changes by $\Delta\ell$.

$$\Delta\ell = \alpha\ell\Delta T$$

where α, the coefficient of linear expansion, is a characteristic of the material. α is measured in units of $1/C^{o}$, (or $1/F^{o}$), ℓ in meters (or any other suitable length unit) and ΔT in C^{o}. A positive ΔT normally producese a positive $\Delta\ell$ and a negative ΔT normally produces a negative $\Delta\ell$.

For example, if a copper rod ($\alpha = 17 \times 10^{-6}/C^{o}$) of length 1.5 meters has its temperature changed from 20 to 100^{o}C, its length will change by

$$\Delta\ell = \alpha\ell\Delta T = (17 \times 10^{-6}/C^{o})(1.5\ m)(100 - 20)C^{o} = 0.0020\ m$$

Table 1 gives values of α for some common materials.

*These values are true at a "standard" atmospheric pressure, 1 atm = 1.013×10^{5} Pa.

Table 1

Coefficients of linear expansion, α

Substance	Coefficient, x $10^{-5}/^{\circ}C$	Coefficient, x $10^{-5}/^{\circ}F$
Aluminum	2.4	1.3
Brass	1.8	1.0
Concrete	0.7-1.2	0.4-0.7
Copper	1.7	0.94
Iron	1.2	0.67
Lead	3.0	1.7
Quartz	0.05	0.008
Silver	2.0	1.1
Steel	1.1-1.2	0.67

A Bimetallic Strip is a device constructed by attaching a thin strip of one metal to a thin strip of another metal (iron and brass are typical metals used). If the combination is straight at one temperature it will bend at lower and higher temperatures. For a strip clamped at one end (a brass-iron strip)

$$\Delta s = (L_o^2/2d)(\alpha_B - \alpha_I)\Delta T$$

where Δs is the distance traveled by free end of strip, L_o is length of strip, 2d is the thickness of strip, ΔT is the temperature change and α_B, α_I are the respective linear expansion coefficients of brass and iron. The deflection Δs is much greater than the linear expansion (contraction) of either material.

Obviously, such a device can be used as a thermometer.

When a volume V is subjected to a temperature change ΔT, its volume changes by ΔV

$$\Delta V = \beta V \Delta T \quad \text{- Liquids}$$
$$\Delta V = (3\alpha)V\Delta T \quad \text{- "Isotropic Solids"}$$

where β, the coefficient of volume expansion, is a characteristic of the liquid, α is the previously defined linear expansion coefficient. β is measured in units of $1/C^{\circ}$ (or $1/F^{\circ}$). A positive ΔT normally produces a positive ΔV and a negative ΔT normally provides a negative ΔV.

For example, if 10 cm^3 of mercury ($\beta = 18 \times 10^{-5}/C^{\circ}$) is heated from $0^{\circ}C$ to $100^{\circ}C$, its volume will change by

$$\Delta V = \beta V \Delta T = (18 \times 10^{-5}/C^{\circ})(10\ cm^3)(100\ C^{\circ}) = 0.18\ cm^3$$

When a solid container holding a liquid is heated (or cooled), both the container and the liquid expand (contract). The net apparent change in the volume of the liquid equals the difference between the two expansions (contractions). That is,

$$\Delta V = (\beta - 3\alpha)V\Delta T$$

In a liquid in glass thermometer a temperature change produces a volume change in the liquid((alcohol + water) or mercury, usually), and a consequent change in the length of the liquid volume in the thermometer stem.

For example, if a temperature change of 10 degrees produces a length change of 0.50 cm, then a change of 50 degrees will produce a change of 2.5 cm.

Table 2 gives values of β for some common liquids.

Table 2

Coefficients of volume expansion, β

Substance	Coefficient, x $10^{-4}/^{o}C$	Coefficient, x $10^{-4}/^{o}F$
Ethyl alcohol	11	6.1
Glass (average)	0.2	0.1
Glycerin	5.1	2.8
Ice	0.5	0.3
Mercury	1.8	1.0
Pyrex glass	0.09	0.05
Water	2.1	1.2

The resistance of a conductor is a function of temperature. If a certain conductor has a resistance R_o at $0^{o}C$ and its temperature is changed by ΔT, then its new resistance R will be

$$R = R_o(1 + k\Delta T)$$

where k, the temperature coefficient of resistance, is measured in $1/C^{o}$, R and R_o are measured in ohms and ΔT is in C^{o}.

In platinum resistance thermometers a temperature change in the wire produces a resistance change in the circuit and a consequent current change which may be read directly as a temperature.

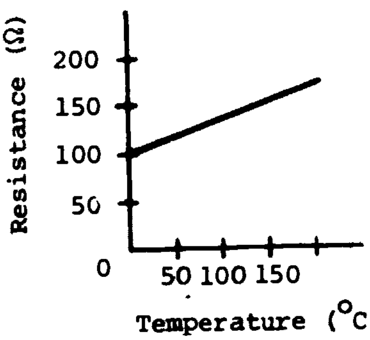
Table 3 gives values of k as well as other useful values for various metals.

Table 3

Properties of Metals as Conductors

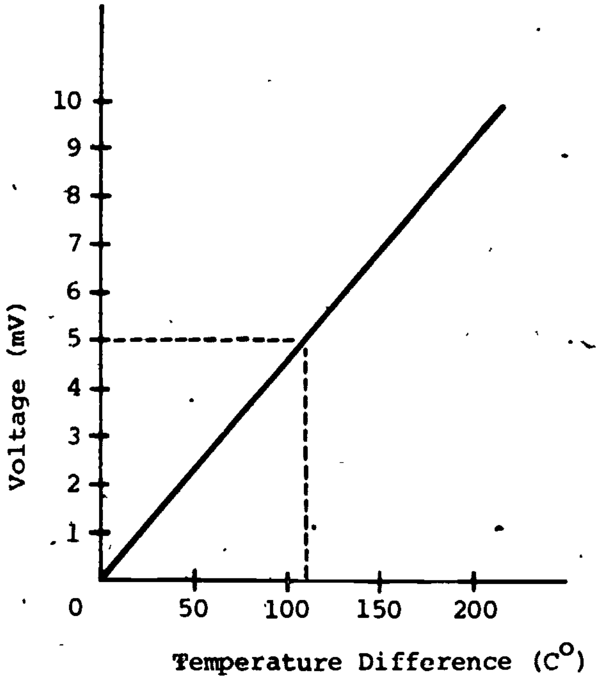
	Resistivity (at 20°C), (ohm-m)	Temperature Coefficient of Resistance k (per C°)	Density (g/cm^3)	Melting Point (°C)
Aluminum	2.8×10^{-8}	3.9×10^{-3}	2.7	659
Copper	1.7×10^{-8}	3.9×10^{-3}	8.9	1080
Carbon (amorphous)	3.5×10^{-5}	-5×10^{-4}	1.9	3500
Iron	1.0×10^{-7}	5.0×10^{-3}	7.8	1530
Manganin	4.4×10^{-7}	1×10^{-5}	8.4	910
Nickel	7.8×10^{-8}	6×10^{-3}	8.9	1450
Silver	1.6×10^{-8}	3.8×10^{-3}	10.5	960
Steel	1.8×10^{-7}	3×10^{-3}	7.7	1510
Wolfram (tungsten)	5.6×10^{-8}	4.5×10^{-3}	19	3400

A typical curve relating resistance to temperature is given below:



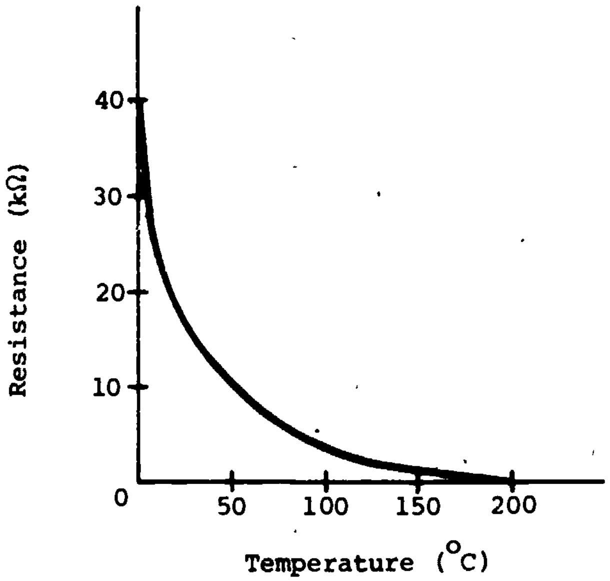
A <u>thermocouple</u> is a device in which two junctions of dissimilar metals (copper and constantan, for example) are kept at different temperatures. When properly connected to a voltage indicating device, a <u>voltage</u> will be developed which is <u>proportional to the temperature difference</u> of the two junctions. One junction is normally kept at a reference temperature of 0°C.

A typical curve describing such a device is given below:



A thermistor is a device whose resistance changes non-linearly with temperature. When connected to a proper electric circuit, such a device can be used to measure temperature.

A typical curve describing such a device is given below:



LABORATORY

The student should be able to use a variety of temperature measuring devices (liquid in glass, thermocouple, thermistor, resistance thermometers, for example) to measure the temperature of various objects (liquids, gases, operating electronic components, operating household appliances, etc.).

SOLVED PROBLEMS

1. Ethyl alcohol freezes at about $-117^{\circ}C$. What are the corresponding Fahrenheit and Kelvin readings?

$$
\begin{aligned}
{}^{\circ}F &= 9/5\left(\left[\text{Temp}\,({}^{\circ}C)\right] + 32\right){}^{\circ}F \\
&= (9/5(-117) + 32)\,{}^{\circ}F \\
&= -179^{\circ}F
\end{aligned}
$$

$$
\begin{aligned}
{}^{\circ}K &= \left[\text{Temp}\,({}^{\circ}C) + 273\right]{}^{\circ}K \\
&= (-117 + 273)\,{}^{\circ}K \\
&= 156^{\circ}K
\end{aligned}
$$

2. A steel tape measure (calibrated at $20^{\circ}C$) measures the length of a copper rod as 100.00 cm at $10^{\circ}C$. What is the actual length of the rod?

The steel tape is short at $10^{\circ}C$

$$\Delta \ell = \alpha \ell \Delta T$$

$$
\begin{aligned}
\frac{\Delta \ell}{\ell} = \alpha T = \left(\frac{11 \times 10^{-6}}{C^{\circ}}\right)(10C^{\circ}) &= 11 \times 10^{-5} \\
&= 1.1 \times 10^{-4} \\
&= 0.00011 \\
&= 0.011\%
\end{aligned}
$$

That is, the tape is short by 0.011%.

Finally, $\frac{99.989\%}{100.00 \text{ cm}} = \frac{100\%}{X}$

$$X = \frac{(100)(100)\text{ cm}}{99.989} = 100.01 \text{ cm}$$

3. The resistance R_{20} of a coil of insulated copper wire is 4.000 Ω at 20°C. Find its resistance R_{200} at 200°C. Such conditions as these might simulate the operation of an electric heating appliance (toaster, space heater, etc.).

$$R_{20} = 4.000\ \Omega = R_o(1 + k\Delta T)$$

$$= R_o\left[1 + (3.9 \times 10^{-3}/C^o)(20C^o)\right]$$

$$= (1.078)R_o$$

$$R_o = \frac{4.000\ \Omega}{1.078} = 3.711\ \Omega$$

$$R_{200} = 3.71\ \Omega\left[1 + (3.9 \times 10^{-3}/C^o)(200C^o)\right]$$

$$= 6.60\ \Omega$$

4. In a typical application of a thermocouple thermometer the voltage developed is about 5 millivolts. The reference junction is kept at 0°C. Find the temperature of the hotter junction.

From graph above (page 62) 5 mV corresponds to about 110-115°C.

STUDENT PROBLEMS

1. At what temperature are the Fahrenheit and Centigrade readings identical? What is the equivalent Kelvin reading?

(-40°C; -40°F; 233°K)

2. An accurately calibrated Pyrex glass vessel (linear expansion coefficient = $3.0 \times 10^{-6}/C^o$) is filled with exactly 1 liter (=1000 cm^3 = $10^{-3}\ m^3$) of mercury (volume expansion coefficient = $1.8 \times 10^{-4}/C^o$) at 20°C. How much mercury will spill over when the temperature is raised to 100°C?

($14\ cm^3$)

3. A thermistor is being used to measure temperature. The thermistor is part of a circuit and the resistance of the circuit, exclusive of the thermistor, is 50 ohms. Assuming that a constant potential is applied to the circuit, by about what per cent will the current in the circuit change if the thermistor temperature goes from 20°C to 100°C?

(about 20% increase in current)

4. A brass-iron bimetallic strip of length 25 cm, thickness 0.5 cm, is straight at 25°C. One end of the strip is clamped and the strip is heated to 425°C. By what distance will the free end of the strip travel?

(3 cm)

SECTION 2 - SPECIFIC HEAT

Heat is a form of energy associated with the random motion of the molecules of a substance. An increased temperature implies an increased random motion

Heat Energy is measured in units of calories (cal), kilocalories (kcal), and British thermal units (Btu).

1 calorie of heat energy is required to raise the temperature of 1 gram of water by 1 celsius degree (specifically from 14.5 to 15.5°C).

$$1 \text{ calorie} = 4.185 \text{ joules}$$

1 kilocalorie (kcal or kg-cal) equals 1000 calories or

$$1 \text{ kilocalorie} = 4185 \text{ joules}$$

1 British thermal unit of heat energy is required to raise the temperature of 1 pound of water by 1 Fahrenheit degree (specifically from 63 to 64°F).

$$1 \text{ Btu} = 778 \text{ ft-lb} = 252 \text{ cal} = 0.252 \text{ kcal}$$

(NOTE: The relationship 1 kcal = 4185 J is sometimes referred to as the mechanical equivalent of heat.

The Heat Capacity of a body is the quantity of heat needed to raise the temperature of the body by one degree. It is measured in kcal/C° or Btu/F°.

The Specific Heat c of a body is the quantity of heat needed to raise unit mass of the body by one degree. It is measured in kcal/(kg C°) or Btu/(lb F°). Numerically, the specific heat has the same value in both systems of units. That is kcal/(kg C°) = Btu/(lb F°).

The relationship between the heat energy Q gained (lost) by a body (mass m and specific heat c) undergoing a temperature change ΔT is

$$Q = mc\Delta T$$

It is assumed that the body does not un[illegible] o a change in state.

For example, if a sample of lead (mass of 1 kg, specific heat of 0.030 kcal/(kg C°)) is to be heated by 50 C°, the required heat energy is $Q = mc\Delta T = (1\text{ kg})(0.030\text{ kcal/(kg C}^\circ))(50\text{ C}^\circ) = 1.5\text{ kcal}$.

When two or more objects with different temperatures are brought into contact with each other energy is conserved and

(Heat energy lost by "hot" objects) = (Heat energy gained by "cool" objects)

The above relationship assumes no heat energy loss (or gain) to (from) the surroundings; it is referred to as the Method of Mixtures.

If a device (mass m, specific heat c) is being heated electrically (assuming 100% efficiency) by a power supply delivering P watts (current I, voltage V) for a time t

$$Pt = IVt = mc\Delta T$$

where ΔT is the temperature change and a proper choice of units must be made. If the heat delivering device obeys Ohm's Law,

$$Pt = IVt = I^2Rt = \frac{V^2}{R}t = mc\Delta T$$

where R is the resistance.

Table 4 gives values of c for some common materials.

Table 4

Specific Heat of Various Materials

Material	Specific Heat (kcal/(kg C^o) or Btu/(lb F^o))
alcohol (ethyl)	0.58
aluminum	0.22
copper	0.093
glass	0.20
ice	0.50
iron	0.11
lead	0.030
mercury	0.033
steam	0.48
water	1.00
wood	0.42
zinc	0.092

LABORATORY

Using the traditionally employed method of mixtures, the student should be able to measure the specific heat of various materials.

Also, using such methods as the continuous flow electric calorimeter and/or methods in which mechanical energy is directly converted into heat energy, the student should be able to measure the mechanical equivalent of heat.

Finally, using traditionally accepted methods the student should be able to measure the average specific heat of such devices as an electric toaster or an electric iron.

SOLVED PROBLEMS

1. How many kilocalories are needed to raise the temperature of 500 grams of aluminum from 20°C to 100°C?

$$Q = mc\Delta T = (0.500\ \text{kg})(0.22\ \text{kcal}/(\text{kg C}^\circ))(100 - 20)\text{C}^\circ$$

$$= 8.8\ \text{kcal}$$

2. An aluminum container of mass 100 grams contains 500 grams of water. Both are at a temperature of 20°C. 500 grams of water at 80°C is then mixed with the cooler water. Find the final equilibrium temperature T of the mixture. Assume no loss to the surroundings.

Heat lost = Heat gained

$$(0.500\ \text{kg})(1\ \text{kcal}/(\text{kg C}^\circ))(80 - T)\text{C}^\circ$$

$$= (0.100\ \text{kg})(0.22\ \text{kcal}/(\text{kg C}^\circ))(T - 20)\text{C}^\circ$$

$$+ (0.500\ \text{kg})(1\ \text{kcal}/(\text{kg C}^\circ))(T - 20)\text{C}^\circ$$

$$40.0 - 0.500\,T = 0.022\,T - 0.44 + 0.500\,T - 10.0$$

$$50.44 = 1.022\,T$$

$$T = 49.4^\circ\text{C}$$

3. A 500 W electric heater is being used to heat 1 kg of water in a 300 g aluminum vessel. What temperature change will be produced in 5 minutes? Assume no heat loss to the surroundings.

Energy in = (Power)(Time)

= (500 W)(300 s) = 150,000 J

= (150,000 J)(1 kcal/4185 J)

= 35.84 kcal

$$\text{Heat} = 35.84\ \text{kcal} = (0.300\ \text{kg})(0.22\ \text{kcal}/(\text{kg C}^\circ))(\Delta T)$$
$$+ (1.00\ \text{kg})(1\ \text{kcal}/(\text{kg C}^\circ))(\Delta T)$$
$$= (0.066 + 1)(\text{kcal}/\text{C}^\circ)(\Delta T)$$
$$\Delta T = 35.84/1.066 = 33.62\ \text{C}^\circ$$

STUDENT PROBLEMS

1. Determine the final temperature when 1 kg of water at 20°C is mixed with 1.5 kg of water at 80°C.

   (56°C)

2. A 400 gram piece of material at 100°C is lowered into a 250 gram aluminum container holding 500 grams of water at 20°C. The final equilibrium temperature of the mixture is 26.9°C. Find the specific heat of the material.

   (0.131 kcal/(kg C°))

3. In an experiment in which water flows by an electrically heated resistor at a known rate the following data are collected:

   Power dissipated in resistor - 500 W
   Temperature of water entering device - 20°C
   Temperature of water leaving device - 30°C
   Total mass of water involved - 7.20 kg
   Time of flow - 10 minutes

   From this data calculate the mechanical equivalent of heat. Compare with the accepted value.

   (4.17×10^3 J/kcal)

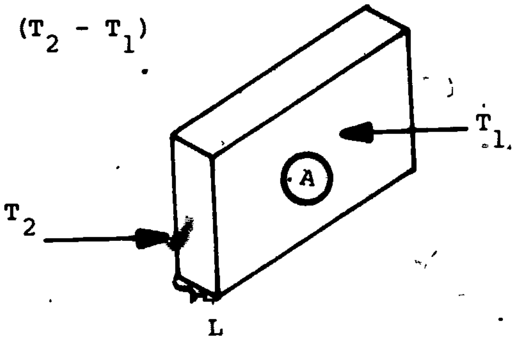
SECTION 3 - TRANSFER OF HEAT

Heat Transfer can be accomplished through the processes of conduction, convection, radiation.

Conduction refers to a process of heat transfer through a solid object or a fluid in which energy is passed along from molecule to molecule but in which there is no net motion of the molecules involved.

For example, for a rectangular slab of thickness L and cross-sectional area A with one side at temperature T_1 and the other side at temperature T_2, $(T_2 > T_1)$ the heat energy transferred per unit time, P, is

$$P = \frac{kA}{L}(T_2 - T_1)$$



k is called the thermal conductivity and is measured in units of W/(m C°) or kcal/(s m C°).

(NOTE: $(T_2 - T_1)L$ is sometimes referred to as the temperature gradient.

Table 5 lists the thermal conductivities of some common materials.

Table 5

Thermal Conductivities of Some Common Materials

Material	Thermal Conductivity (kcal/(m s C°))	(W/m-C°)
Copper	0.092	3.9×10^{2}
Aluminum	0.051	2.1×10^{2}
Iron, Steel	0.0011	4.6
Ice	5.2×10^{-4}	2.2
Glass	1.9×10^{-4}	0.80
Wood (Oak, Pine)	0.38×10^{-4} 0.28×10^{-4}	0.16 0.12
Rock and Glass Wool	0.093×10^{-4}	0.039
Water	1.4×10^{-4}	0.59
Hydrogen	0.41×10^{-4}	0.17
Air	0.055×10^{-4}	0.023

Convection is a process of heat transfer in which heat energy is carried along by a moving fluid such as air or water.

* By analogy with the conduction case the heat energy transferred per unit time, P, by a moving current of fluid with cross-section A, velocity v, thickness (or length) L and a temperature gradient $(T_2 - T_1)/L$, is

$$P = \frac{kA\sqrt{v}}{B} (T_2 - T_1)$$

where $L = B/\sqrt{v}$ and B is a constant that depends upon the properties of the fluid such as its specific heat and viscosity.

Radiation is a process of heat transfer in which the energy is transferred by means of electromagnetic waves.

For a body of surface area A and temperature $T(^{\circ}K)$, the heat energy radiated per unit time is

$$P = \varepsilon\sigma AT^4$$

where P is power; σ, the Stefan-Boltzmann constant, is 5.67×10^{-8} $W/(m^2\text{-}(K^{\circ})^4)$ and ε is the emissivity (a pure number having no units) which depends upon the surface of the radiating body. ε is always between 0 and 1. $\varepsilon = 1$ for a black body (it looks black at room temperature). A black body is the "perfect" radiator (and absorber).

The above relation considers only radiation emitted by the body itself. The body, of course, absorbs radiation from the outside and the net radiation emitted is

$$P \cong \varepsilon\sigma AT^3\Delta T$$

where T is the temperature of the body and ΔT is the temperature difference between the body and its surroundings.

LABORATORY

Using commercially available equipment the student should be able to measure the thermal conductivity of good conductors like copper and poor conductors like wood or cork as well as study the phenomena of temperature lag of a cooling or warming body such as a calorimeter or a thermometer.

*Optional material

SOLVED PROBLEMS

1. An oak entry door to a house is 10 cm thick, 250 cm high and 100 cm wide. How much heat energy will be transferred through this door in 8 hours if the average outside temperature is 0^oC and the average inside temperature is 25^oC?

$$P = \frac{kA}{L}(T_2 - T_1)$$

$$= \frac{(0.16\ W/m\ C^o)(2.50\ m)(1.00\ m)}{0.100\ m}(25-0)C^o$$

$$= 100\ W \qquad \text{(2 significant figures)}$$

Heat energy through door in 8 hours is

$$(100\ W)(8\ hr)\left(\frac{3600\ s}{1\ hr}\right) = 2.9 \times 10^6\ J$$

$$= 6.9 \times 10^2\ kcal$$

2. A typical incandescent light bulb filament (tungsten) has an effective area of $7.70 \times 10^{-5}\ m^2$ and operates at a temperature of 2450^oK. The emissivity of tungsten is about 0.30. Find the energy radiated in 1 hour.

$$P = \varepsilon\sigma AT^4$$

$$= (0.30)(5.67 \times 10^{-8}\ W/m^2(K^o)^4)(7.70 \times 10^{-5}m^2)(2450^oK)^4$$

$$= 47.2\ W$$

Energy radiated in 1 hour

$$= (47.2\ W)(3600\ s) = 1.70 \times 10^5\ J$$

(NOTE: Strictly speaking, answers should have only one significant figure.)

STUDENT PROBLEMS

1. A typical double boiler has an aluminum vapor pan, diameter of 25 cm and thickness of 2 mm. The lower pan contains water at 90^oC and the upper pan contains milk at 5^oC. Find the rate at which heat is transferred to the milk.

(1.1×10^2 kcal/s)

2. The human body has a normal temperature of about 98.6°F (37°C). At what rate does it radiate per unit area? If you are in a 70°F (21°C) room what is your <u>net</u> radiation per unit area? Assume an emissivity of about 0.1.

(5×10^1 W/m^2; 3 W/m^2)

END OF CHAPTER PROBLEMS

1. Air conditioners are sometimes rated in tons; 1 ton means the air conditioner can remove 12,000 Btu/hr from the space to which it is attached. 1 Btu/hr = 7×10^{-5} kcal/s. Suppose that a 1 ton air conditioner can maintain an empty room at 70°F (21°C) when the outside temperature is 95°F (35°C) by running half of the time. How many people can occupy the room without exceeding the capacity of the air conditioner? Each person liberates about 500 Btu/hr (about 0.035 kcal/s).

(12 people)

2. In the winter why does the metal blade of a snow shovel feel colder than the wooden handle of the shovel?

3. The sun has a diameter of about 1.39×10^6 m and an average temperature of about 6000°K. Assuming that the sun is a perfect black body how much power is radiated? (Area of a sphere of radius R is $4\pi R^2$.)

(4.46×10^{20} W)

CHAPTER V

PROPERTIES OF GASES AND LIQUIDS

SECTION 1 - DENSITY, SPECIFIC GRAVITY, ARCHIMEDES PRINCIPLE

The density ρ of a body is defined as

$$\rho = \text{density} = \text{mass per unit volume}$$

$$= \frac{\text{mass of body}}{\text{volume of body}} = \frac{m}{V}$$

Typical density units are g/cm^3, kg/m^3, and g/liter.

The density of water (at about 4^oC) is $1.000\ g/cm^3 = 1000\ kg/m^3$.

The weight density ρ_g of a body is defined as

$$\text{Weight density} = \rho_g = \text{weight per unit volume}$$

$$= \frac{\text{weight of body}}{\text{volume of body}} = \frac{W}{V}$$

Typical units are N/m^3, lb/ft^3.

The weight density of water (at about 4^oC) is $62.4\ lb/ft^3$ or $9800\ N/m^3$.

Specific Gravity is defined as

$$\text{S.G.} = \text{Specific Gravity} = \frac{\text{density of body}}{\text{density of water}}$$

which can be shown to give

$$\text{S.G.} = \frac{\text{mass of body}}{\text{mass of equal volume of water}}$$

$$= \frac{\text{weight of body}}{\text{weight of equal volume of water}}$$

Specific gravity is a pure number, independent of any unit system.

For example, the density of aluminum is $2.70\ g/cm^3$ or $2700\ kg/m^3$ while its specific gravity is merely 2.70. Similarly, the weight density of aluminum is $168\ lb/ft^3$ or $26,460\ N/m^3$.

A table of values of density, weight density and specific gravity of several typical solids, liquids and gases is given below.

TABLE

Substance	Density (g/cm^3)	Density (kg/m^3)	Weight Density (N/m^3)	Weight Density (lb/ft^3)	Specific Gravity
Aluminum	2.70	2700	26,460	168	2.70
Brass	8.70	8700	85,300	5430	8.70
Copper	8.92	8920	87,400	556	8.92
Iron	7.86	7860	77,000	490	7.86
Lead	11.3	11300	111,000	705	11.3
Water (4°C)	1.000	1000	9,800	62.4	1.000
Alcohol (Ethyl)	0.801	801	7,860	50	0.801
Mercury	13.6	13600	133,000	849	13.6
Air *	0.00129	1.29	12.6	0.080	0.00129
Helium *	0.00018	0.18	1.76	0.011	0.00018
Oxygen *	0.00143	1.43	14.01	0.089	0.00143

*1 atmosphere, 0°C = standard temperature and pressure (STP)

- Archimedes Principle states that a body wholly or partly immersed in a fluid (liquid or gas) is buoyed up by a force equal to the weight of the fluid displaced by the body.

- Buoyancy (up force) $= F_B =$ Weight of fluid displaced

$$= W_F$$
$$= \rho_F V_F g,$$

where ρ_F = density of fluid displaced, V_F = volume displaced and $g = 9.8\ m/s^2$.

For example, a helium filled balloon which displaces 0.100 m^3 of air (density of 1.29 kg/m^3) will feel an upward buoyant force of $(1.29\ kg/m^3)(0.100\ m^3)(9.8\ m/s^2) = 1.26$ N.

- For solids with a density greater than water

$$\text{Specific Gravity} = \frac{\text{weight in air}}{(\text{weight in air})-(\text{apparent weight in water})}$$

NOTE: The buoyant effect of the air is neglected here since it is very small compared to the buoyant effect of water.

One method used to measure the specific gravity of liquids involves measuring the mass of a solid in air, when submerged in water and when submerged in the liquid. Here

$$\text{Specific Gravity of liquid} = \frac{\text{mass of displaced liquid}}{\text{mass of equal volume of water}}$$

NOTE: Again we neglect buoyancy of air effect.

LABORATORY

Using a variety of length measuring devices (meter stick, vernier caliper, micrometer caliper, etc.) a variety of mass measuring devices (spring balances, double pan balances, substitution type balances, etc.) as well as graduated cylinders, specific gravity bottles, etc., a student should be able to measure the density, weight density and specific gravity of various solids and liquids.

WORKED EXAMPLES (Neglect buoyancy of air.)

1. Find the density ρ, the weight density ρ_g and the specific gravity of a sample of iron ore if $5.50\ m^3$ has a mass of 38,500 kg.

$$\text{density} = \rho = \frac{38{,}500\ \text{kg}}{5.50\ \text{m}^3} = 7000\ \frac{\text{kg}}{\text{m}^3} = 7.00 \times 10^3\ \text{kg/m}^3$$

$$\text{weight density } \rho_g = (7000\ \frac{\text{kg}}{\text{m}^3})(9.8\ \text{m/s}^2) = 68{,}600\ \frac{\text{N}}{\text{m}^3} = 6.86 \times 10^4\ \text{N/m}^3$$

$$\text{Specific Gravity} = \text{S.G.} = \frac{\text{density of iron}}{\text{density of water}}$$

$$= \frac{7000\ \text{kg/m}^3}{1000\ \text{kg/m}^3}$$

$$= 7.00$$

2. A helium filled balloon rises when released because the upward buoyant force on the balloon exceeds the downward force of gravity on the balloon and its contents. Helium is a "lighter" substance than air. How large a balloon will be needed to support a 150 lb person? (Neglect the weight of the balloon and helium.)

The upward buoyant force F_B must be at least 150 pounds. The volume of air V_F which will have this weight is given by

$$F_B = \rho_F V_F g$$

$$V_F = \frac{F_B}{\rho_F g} = \frac{(150\ \text{lb})(4.45\ \text{N}/1.00\ \text{lb})}{(1.29\ \text{kg/m}^3)(9.8\ \text{m/s}^2)} = 52.8\ \text{m}^3$$

3. An object has a mass of 71 g in air and apparent masses of 43 g in water and 20 g in sulfuric acid. Find the specific gravity of the acid.

$$\text{S.G.} = \frac{\text{mass of displaced acid}}{\text{mass of equal volume of water}}$$

$$= \frac{\text{apparent loss of weight in acid}}{\text{apparent loss of weight in water}}$$

$$= \frac{(71 - 20)\ \text{g}}{(71 - 43)\ \text{g}} = \frac{51}{28} = 1.82$$

4. A sample has a mass of 50 g in air and an apparent mass of 40 g in water. Find its specific gravity.

$$\text{S.G.} = \frac{\text{weight in air}}{\text{apparent weight loss in water}} = \frac{50\ \text{g}}{10\ \text{g}} = 5.00$$

STUDENT PROBLEMS

1. Find the weight (in pounds) of 4 ft^3 of water, aluminum, mercury.

   (250 lb; 672 lb; 3.40×10^3 lb)

2. A typical circular backyard swimming pool has a diameter of 15 feet and a depth of 4 feet. Find the weight of the water in this pool. Compare this to the weight of an automobile.

   (4.41×10^4 lb; about 15 autos have same weight)

NOTE: Volume of cylinder = $(\frac{\pi D^2}{4})h$

where D = diameter
h = depth (height)

3. A solid measures 1000 g in air and 600 g in a liquid whose specific gravity is 0.700. Find the specific gravity of the solid.

   (1.75)

4. A metal sphere has a diameter of 10 cm and a mass of 4,000 g. Find its density in g/cm^3, in kg/m^3. (Volume of sphere equals $4/3\pi R^3$; where R = radius.)

   ($7.64\ g/cm^3$; $7.64 \times 10^3\ kg/m^3$)

SECTION 2 - PHASE CHANGES

Any substance which has a definite chemical composition will exist in one of four phases; solid, liquid, gas, or plasma.

The heat of fusion H_f of a solid is the amount of heat energy Q needed to change a unit mass of the solid into a unit mass of liquid without a change in temperatuare.

Heat of fusion of ice = 80 kcal/kg, (at $0^{\circ}C$ and 1 atmosphere of pressure)

= 144 Btu/lb (at $32^{\circ}F$ and 1 atmosphere of pressure)

Example.

How much heat Q is needed to melt 20 grams of ice at $0^{\circ}C$?

$$Q = mH_f$$

$$= (0.0200 \text{ kg})(80 \text{ kcal/kg}) = 1.60 \text{ kcal}$$

The heat of vaporization H_v of a liquid is the amount of heat energy Q needed to change a unit mass of the liquid into a unit mass of gas without a change in temperature.

Heat of vaporization of water = 540 kcal/kg, (at $100^{\circ}C$ and 1 atmosphere of pressure)

= 972 Btu/lb, (at $212^{\circ}F$ and 1 atmosphere of pressure)

Example.

How much heat Q must be removed from 20 g of water vapor or steam at $100^{\circ}C$ to condense it to liquid at the same temperature?

$$Q = mH_v$$

$$= (0.0200 \text{ kg})(540 \text{ kcal/kg})$$

$$= 10.8 \text{ kcal}$$

A plasma occurs when enough heat has been added to a gas to cause the molecules to break up into pairs of electrically charged particles called ions.

NOTE: Students should review the material of Chapter IV which deals with specific heat and the method of mixtures.

LABORATORY

Students should be able to use thermometers, calorimeters, steam traps, boilers, thermistors, etc. to measure heats of fusion and vaporization.

WORKED EXAMPLES

1. Find the final equilibrium temperature T when 200 g of ice at $0^{o}C$ and 400 g of water at $50^{o}C$ are mixed together.

Heat gained by ice = Heat lost by warm water

$$(0.200 \text{ kg})(80 \text{ kcal/kg}) + (0.200 \text{ kg})(1 \text{ kcal/(kg C deg)})(T-0)C^{o}$$

$$= (0.400 \text{ kg})(1 \text{ kcal/(kg } C^{o}))(50-T)C^{o}$$

or $16.0 + 0.200\,T = 20.0 \quad 0.400\,T - 20.0$

$$0.600\,T = 4.0$$

$$T = 6.7^{o}C$$

2. A vessel contains 500 g of water and 300 g of ice - equilibrium temperature of $0^{o}C$. 100 g of steam at $100^{o}C$ is introduced into the mixture. Describe what happens.

Heat lost by steam = Heat gained by ice and water

$$(0.100 \text{ kg})(540 \text{ kcal/kg}) + (0.100 \text{ kg})(1 \text{ kcal/(kg } C^{o}))(100-T)C^{o}$$

$$= (0.300 \text{ kg})(80 \text{ kcal/kg}) + (0.300 \text{ kg})(1 \text{ kcal/(kg } C^{o}))(T-0)C^{o}$$

$$+ (0.500 \text{ kg})(1 \text{ kcal/(kg } C^{o}))(T-0)C^{o}$$

$$54.0 + 10.0 - 0.100\,T = 24.0 + 0.300\,T + 0.500\,T$$

$$40.0 = 0.900\,T$$

$$T = 44.4^{o}C$$

You have 900 g of water at $44.4^{o}C$.

STUDENT PROBLEMS

1. How much heat is absorbed when 140 g of ice ($0^{o}C$) is melted?

(11.2 kcal)

2. 0.500 kg of water is at 20°C. How much heat is needed to boil it all away?

(310 kcal)

3. A student takes the following data in a lab situation measuring the heat of fusion of ice (ice is at 0°C):

calorimeter mass	= 60 g
water in calorimeter	= 400 g
total mass of calorimeter,	
water and ice added	= 618 g
initial temp. of water	= 38°C
equilibrium temp.	= 5°C
c(calorimeter)	= 0.10 cal/g°C

Find the heat of fusion of ice.

79.8 kcal/kg)

SECTION 3 - PRESSURE AND ITS MEASUREMENT

$$\text{Pressure} = \frac{\text{force}}{\text{area}} \qquad P = \frac{F}{A}$$

NOTE: Force F must be perpendicular to area A.

Units of pressure = $\frac{lb}{ft^2}$, $\frac{lb}{in^2}$, $\frac{N}{m^2}$, $1\ \frac{N}{m^2}$ = 1 pascal (Pa)

Example.

Find the pressure on the bottom of a cubical container, 1 ft on a side, of water.

The total force on the bottom of the container is 62.4 lb (water weighs 62.4 lb/ft^3). The total surface area of the bottom of the container is (1 ft)(1 ft) = 1 ft^2, or 144 in^2.

$$P = \frac{F}{A} = \frac{62.4\ lb}{144\ in^2} = 0.433\ \frac{lb}{in^2} = 62.4\ \frac{lb}{ft^2}$$

The pressure P due to any column of fluid of height h and density ρ is

$$P = \rho g h$$

where g = acceleration of gravity.

Pressure depends only on the depth of the fluid and not the shape of the container.

Example.

What is the pressure (in lb/in^2) at the bottom of a 34 foot vertical water pipe?

$$P = \rho gh = (62.4 \frac{lb}{ft^3})(34\ ft) = (2121.6 \frac{lb}{ft^2})(\frac{1\ ft^2}{144\ in^2}) = 14.7 \frac{lb}{in^2}$$

Atmospheric Pressure. Air exerts a pressure on any object immersed in it. This pressure varies somewhat depending upon one's location on the surface of the earth. It drops as one goes above the surface of the earth. By convention and measurement the standard sea level value is

$$\begin{aligned} 1 \text{ atmosphere (atm)} &= 14.70\ lb/in^2 \\ &= 1.013 \times 10^5\ N/m^2 \\ &= 1.013 \times 10^5 \text{ Pa (pascals)} \\ &= 1013 \text{ millibars} \\ &= 76 \text{ cm of mercury} \end{aligned}$$

Pascal's Principle: Pressure changes applied to one part of an enclosed fluid (liquid or gas) are transmitted undiminished to all other parts of the enclosed fluid.

Example.

Find the total pressure P_{tot} at a depth of 20 cm in an open container of water. Total pressure is equal to atmospheric pressure P_{atm} plus pressure due to the water P_{H_2O}.

$$\begin{aligned} P_{tot} &= P_{atm} + P_{H_2O} \\ &= 1.013 \times 10^5 \frac{N}{m^2} + \rho gh \\ &= 1.013 \times 10^5 \frac{N}{m^2} + (10^3 \frac{kg}{m^3})(9.8 \frac{m}{s^2})(0.200\ m) \\ &\simeq 1.033 \times 10^5\ N/m^2 \end{aligned}$$

An hydraulic jack, lift or press consists of two connected cylinders - one has a very small cross-sectional area a and the other a very large cross-sectional area A.

A weight placed on the larger piston can be lifted by applying a force on the small piston. According to Pascal's Principle

$$P_{\text{small piston}} = P_{\text{large piston}} = P$$

$$P = \frac{\text{force on small piston (f)}}{\text{area of small piston (a)}} = \frac{\text{weight on large piston (W)}}{\text{area of large piston (A)}}$$

$$\frac{f}{a} = \frac{W}{A}$$

Example.

How much force f must be exerted on the small piston of a hydraulic lift in order to raise an automobile weighing 16,000 N? The areas of the pistons are 4 cm^2 and 1000 cm^2.

$$f = \frac{a}{A} W = \left(\frac{4\ cm^2}{1000\ cm^2}\right)(16{,}000\ N) = \left(\frac{4}{1000}\right)(16{,}000\ N)$$

$$= 64.0\ N$$

LABORATORY

The student should be able to use pressure measuring devices and should be able to verify Pascal's Principle on an hydraulic device.

WORKED EXAMPLES

1. An open container filled with a fluid of density ρ was fitted with a tight fitting piston of area A. Another force F was applied to the piston. Find the pressure at a depth h below the surface.

$$P = P_1 + \rho gh$$

$$= \frac{F}{A} + \rho gh$$

If the force F was increased, then the change in pressure would be transmitted without loss throughout the whole confined liquid.

STUDENT PROBLEMS

1. What is the pressure at the base of a column of mercury 760.0 mm high if ρ for mercury = 13.60×10^3 kg/m^3?

(1.013×10^5 N/m^2)

2. A ballerina weighing 105 lbs stands on one toe. The area of contact between her toe and the floor is 2.5 in^2. What pressure does she exert on the floor?

(42 $lbs/in^2 \cong 3$ atm)

3. An hydraulic barber chair has a large cylinder with an area of 120 cm^2 and a small cylinder with an area of 4 cm^2. How much downward force must be applied on the small cylinder to lift the chair and customer (total weight 250 lbs)?

(8.33 lbs)

4. What is the total pressure on a swimmer who is 1.50 m below the surface of the sea? (Density of sea water is 1025 kg/m^3.)

(1.16×10^5 N/m^2 = 16.9 lb/in^2 = 1.15 atm)

SECTION 4 - GAS LAWS

Boyle's Law: Liquids and solids are considered to be virtually incompressible. However, gases undergo volume changes with changes in pressure (constant temperature). Boyle's Law states:

(Pressure)(Volume) = Constant, (for constant temperature)

or $$P_1V_1 = P_2V_2 \qquad (T = \text{constant})$$

where P_1 and V_1 are the initial pressure and volume and P_2 and V_2 are the final pressure and volume.

Example.

1500 cm^3 of air is contained in a cylinder at 10^5 N/m^2. If this air is now compressed to 300 cm^3 without a change in temperature what will be the final pressure?

$$P_1V_1 = P_2V_2$$

$$P_2 = P_1\frac{V_1}{V_2} = (10^5\ \frac{N}{m^2})(\frac{1500\ cm^3}{300\ cm^3}) = (10^5\ \frac{N}{m^2})(\frac{15}{3}) = 5.00 \times 10^5\ \frac{N}{m^2}$$

Gay Lussac's Law states that the volume of an enclosed gas at constant pressure is directly proportional to its temperature (^oK), or

$$V = kT, \text{ (for constant pressure)}$$

where k is a constant of proportionality

or

$$k = \frac{V}{T}$$

or

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \qquad (P = \text{constant})$$

Example

A container is filled with 10 m^3 of gas at 27°C. The temperature is then raised to 87°C while the volume increases in order to maintain equal pressure. Find this new volume.

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$V_2 = \frac{V_1}{T_1} T_2 = \frac{(10\ m^3)(87^\circ + 273^\circ)K}{(27^\circ + 273^\circ)K} = \frac{(10\ m^3)(360)}{(300)} = 12.0\ m^3$$

Charles' Law states that pressure of a confined gas at constant volume is directly proportional to the temperature, where T is in degrees Kelvin.

$$P = kT \qquad k = \frac{P}{T}\ (\text{constant volume})$$

(k is a proportionality constant.)

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \qquad (V = \text{constant})$$

Example.

A balloon is inflated to a pressure of 3 x 10^5 N/m^2. Its temperature is 7°C. The gas is heated to 35°C. What is the new pressure, assuming volume remains constant?

$$P_2 = P_1 \frac{T_2}{T_1} = \left(\frac{3 \times 10^5\ N}{m^2}\right)\left(\frac{35^\circ C + 273)K}{7^\circ C + 273)K}\right) = \left(\frac{3 \times 10^5\ N}{m^2}\right)\left(\frac{308}{280}\right) = 3.30 \times 10^5\ \frac{N}{m^2}$$

The General Gas Law states that the product of pressure and volume is directly proportional to the Kelvin temperature.

$$PV = kT \qquad \text{or} \qquad k = \frac{PV}{T}$$

where k is a proportionality constant.

or

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

Example.

Nitrogen is stored in tanks at 227°C and 10^5 N/m^2 pressure. If 5 liters at this temperature and pressure are put into a 1 liter tank at 27°C, what is the pressure in the tank?

$$\frac{P_2V_2}{T_2} = \frac{P_1V_1}{T_1}$$

$$P_2 = P_1 \left(\frac{V_1}{V_2}\frac{T_2}{T_1}\right)$$

$$= \left(\frac{10^5\ N}{m^2}\right)\left(\frac{5}{1}\right)\left(\frac{300}{500}\right) = 3.00 \times 10^5\ \frac{N}{m^2}$$

Gauge Pressure. Most pressure measuring gauges give readings which indicate the difference between the actual pressure on a gas and the atmospheric pressure. For example, if an automobile tire gauge reads 30 "pounds", this means that the pressure in the tire is actually about 45 lb/in^2.

LABORATORY

The student should be able to verify the proportionality of pressure to temperature and find the absolute zero point by extrapolation.

Also, the student should be able to verify Boyle's Law and Charles' Law.

STUDENT PROBLEMS

1. 3 liters of hydrogen is at 26.8°C and 1 atmosphere of pressure. It then is compressed to 2 liters at a pressure of 1.80 atmosphere. Find the new temperature of the gas.

   (360°K)

2. An automobile tire when cold (27°C) has a pressure of 45 lbs/in^2 (30 lbs/in^2 + atmospheric). After being driven 100 miles, the temperature of the enclosed air has risen to 52°C. What is the new pressure?

   (48.8 lbs/in^2)

3. A bubble rises from the bottom of a lake where the water alone produces a pressure of 2 atmospheres. If the volume of the bubble is 15 cm^3 at the bottom, what is its volume when it reaches the surface, assuming no temperature change?

   (45 cm^3)

SECTION 5 - HYDRODYNAMICS

Discharge Rate Q is the volume V per second of a fluid that flows through a full pipe of cross-sectional area A.

$$Q = \frac{V}{t} = Av, \text{ where } v = \text{velocity of the fluid.}$$

When there are different cross-sectional areas (A_1, A_2, A_3, ...) in the pipe,

$$A_1v_1 = A_2v_2 = A_3v_3 = \ldots$$

Example.

One end of a circular pipe has a radius r_1 of 3 in. while the other end has a radius r_2 of 6 in. The velocity v_1 of the fluid in the 3 inch section is 6 ft/s. Find the velocity v_2 in the 6 inch section.

$$A_2v_2 = A_1v_1$$

$$v_2 = \frac{A_1}{A_2} v_1 = \frac{\pi r_1^2}{\pi r_2^2} v_1 = \frac{(3 \text{ in})^2}{(6 \text{ in})^2} \left(6 \frac{\text{ft}}{\text{s}}\right) = \frac{9}{36} \left(6 \frac{\text{ft}}{\text{s}}\right) = 1.50 \frac{\text{ft}}{\text{s}}$$

Torricelli's Theorem states that the velocity of outflow of a fluid from a container of the fluid filled to a distance h above the opening will have the same speed as if the liquid had fallen from the same height. That is,

$$v = \sqrt{2gh}$$

Example.

A tank of water (height = 10 ft) is sitting on the ground. A valve is opened 1 ft above the ground. How fast will the water flow from the opening?

$$\begin{aligned} v &= \sqrt{2gh} \\ &= \sqrt{2 \times (32 \text{ ft/s}^2)(10 \text{ ft} - 1 \text{ ft})} \\ &= \sqrt{576 \text{ ft}^2/\text{s}^2} \\ &= 24.0 \text{ ft/s} \end{aligned}$$

Bernoulli's Theorem states that the work done in transportating an incompressible fluid through a frictionless pipe is equal to the change in the total mechanical energy of the fluid.

$$(P_1 - P_2)\frac{m}{\rho} = \frac{1}{2} mv_2^2 + mgh_2 - (\frac{1}{2} mv_1^2 + mgh_1)$$

or

$$P_1 \frac{m}{\rho} + \frac{1}{2} mv_1^2 + mgh_1 = P_2 \frac{m}{\rho} + \frac{1}{2} mv_2^2 + mgh_2$$

where m is the mass of the fluid, ρ is its density, P_1, v_1 and h_1 are the pressure, speed and height of the fluid at one point in the stream; and P_2, v_2 and h_2 are the pressure, speed and height at some other point in the stream. h_1 and h_2 are measured with respect to some arbitrarily chosen level.

Stated in other ways, along a stream of incompressible fluid

$$P \frac{m}{\rho} + \frac{1}{2} mv^2 + mgh = \text{constant} \qquad \text{(energy form)}$$

$$P + \frac{1}{2} \rho v^2 + h\rho g = \text{constant} \qquad \text{(pressure form)}$$

$$\frac{P}{\rho g} + \frac{v^2}{2g} + h = \text{constant} \qquad \text{(height form)}$$

Example.

Fluid flows through a horizontal pipe at a rate of 6 ft^3/s at a place where the cross-sectional area A_1 is 0.200 ft^2 and the pressure P_1 is 30 lb/in^2. What would be the pressure P_2 at a point where the cross-sectional area A_2 is 0.150 ft^2?

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho gh_2$$

Now, $h_1 = h_2$ and $\rho gh_1 = \rho gh_2$ and $v_1 = \frac{Q}{A_1}$ $v_2 = \frac{Q}{A_2}$

$$\therefore \quad P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$$

$$P_2 = P_1 + \frac{1}{2} \rho v_1^2 - \frac{1}{2} \rho v_2^2$$

$$= P_1 + \frac{\rho}{2}(v_1^2 - v_2^2) = P_1 + \frac{\rho}{2} \left[(Q/A_1)^2 - (Q/A_2)^2\right]$$

$$= (30 \frac{lb}{in^2})(\frac{144\ in^2}{ft^2}) + \left[\frac{(62.4\ lb/ft^3)/(32\ ft/s^2)}{2}\right]$$

$$\times \left[(\frac{6\ ft^3/s}{0.200\ ft^2})^2 - (\frac{6\ ft^3/s}{0.150\ ft^2})^2\right]$$

$$= 4320 \frac{lb}{ft^2} + 0.975 \frac{lb\ s^2}{ft^4} (900 \frac{ft^2}{s^2} - 1600 \frac{ft^2}{s^2})$$

$$= 4320 \frac{lb}{ft^2} + 0.975 \frac{lb}{ft^2} (-700)$$

$$= 4320 \frac{lb}{ft^2} - 682.5 \frac{lb}{ft^2}$$

$$= 3637.5 \frac{lb}{ft^2}$$

$$= 3.64 \times 10^3 \text{ lb/ft}^2 = 25.3 \text{ lb/in}^2$$

The <u>Work Done</u> in forcing a volume of fluid through a pipe against an opposing pressure is given by

$$\text{Work} = (\text{average pressure})(\text{volume})$$

$$W = PV$$

<u>Example.</u>

What would be the work done in forcing 100 ft^3 of water into a water main against a pressure of 20 lb/in^2?

$$W = PV$$

$$= (20 \frac{lb}{in^2}) (\frac{144\ in^2}{ft^2}) (100\ ft^3)$$

$$= 2.88 \times 10^5 \text{ ft lb}$$

LABORATORY

The student should be able to use compression balances, spring balances, platform scales, pressure gauges such as Bourdon gauges, hydraulic jacks, hydraulic valves, and volume measuring devices and thus study various aspects of moving fluids.

STUDENT PROBLEMS

1. A siphon is a hose or tube that can be used to move a fluid over some obstruction from one container to another at a lower level. If the lower end of a siphon is 2 ft below the level of the fluid being siphoned, find the rate of flow of the fluid out of the siphon.

   (11.3 ft/s)

2. A tank holding gasoline has a 1 cm^2 hole punched in it 2.50 m below the level of the liquid in the tank. What is the rate at which this gasoline will flow out of the tank? (0.700 liters/s)

3. Water is pumped in an irrigation system so that it causes a 4 in. diameter horizontal pipe to remain full at an average water velocity of 10 ft/s. How many cubic foot per second goes through the pipe? (0.873 ft^3/s)

4. How long would it take the pump in #3 to pump 1 acre-foot of water. 1 acre-foot means that an area of one acre will be covered with 1 foot of water. 1 acre foot = 43,560 ft^3. (13.9 hours)

5. Two pipes at the same horizontal level are 6 in. in diameter and 2 in. in diameter. These pipes are connected. Water flows through the 6 in. section at a rate of 2 ft/s with a pressure of 15 lb/in^2. Find the velocity and the pressure where the diameter drops to 2 in. (18.0 ft/s; 1.85 x 10^3 lb/ft^2)

END OF CHAPTER PROBLEMS

1. A solid metal ball with a specific gravity 7.7 weighs 5 lb in air, and 4.55 lb when immersed in a liquid. Find the specific gravity of the liquid.

   (0.69)

2. How much heat is needed to change 150 g of ice at -40° F into 140 g of steam at 300° F? (28.1 kcal)

3. 400 g of water and 100 g of ice are mixed together in a container and are in equilibrium at 0° C. 300 g of steam at 100° C are fed into the mixture. Find the final equilibrium temperature. Describe the final mixture.

   ((a) 100° C; 193 g of steam and 600 g of water, all at 100° C)

4. A typical backyard above-ground swimming pool has a diameter of 15 feet and a depth of 4 feet. (a) Find the total pressure on the bottom of the pool. (b) Find the total force acting on the bottom of the pool. (c) What is the weight of the water in the pool? (d) Are the answers to (c) and (b) different? Why?

   (16.4 lb/in^2; (b) 4.17 x 10^5 lb; (c) 1.10 x 10^4 lb; (d) Yes)

5. The maximum depth to which scuba divers may safely descend is usually considered to be about 135 feet. (a) What total pressure exists at this level? Air at normal atmospheric pressure is about 20% oxygen and 80% nitrogen. Assume that the volume of air breathed per breath at a depth of 135 feet is the same as that on the surface (sea level); also assume that air enters the lungs at the pressure of the surroundings when one is below the water. Assume a constant temperature. (b) Comment on the effects of breathing air at this depth from a tank filled on the surface with "normal air".

   ((a) 73.2 lb/in^2 or about 5 atm; (b) equivalent to breathing 10% oxygen at the surface)

6. What horsepower is required to pump water to a height of 20 feet and then force it into a main at a pressure of 25 lb/in^2 if 150 ft^3/minute is to be pumped?

   (22.2 hp)

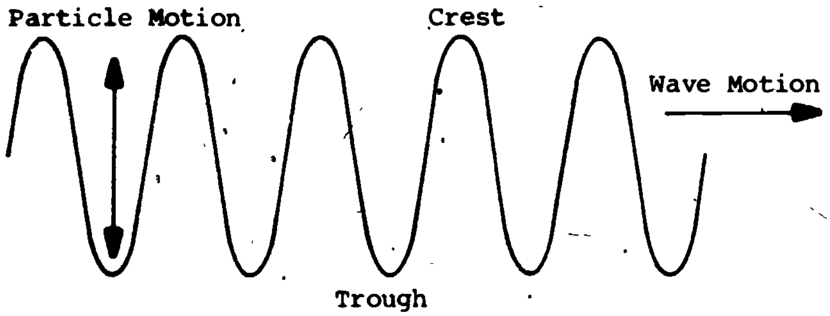
CHAPTER VI

SOUND & WAVE MOTION

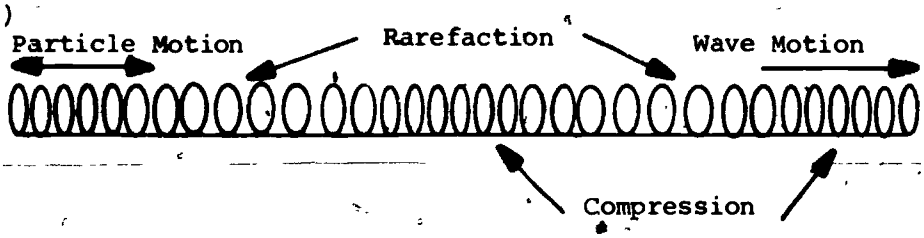
SECTION 1 - BASIC WAVE PROPERTIES

Wave motion involves the transport of energy through a medium, such that there is no mean displacement of the particles of the medium, which vibrate about an equilibrium position.

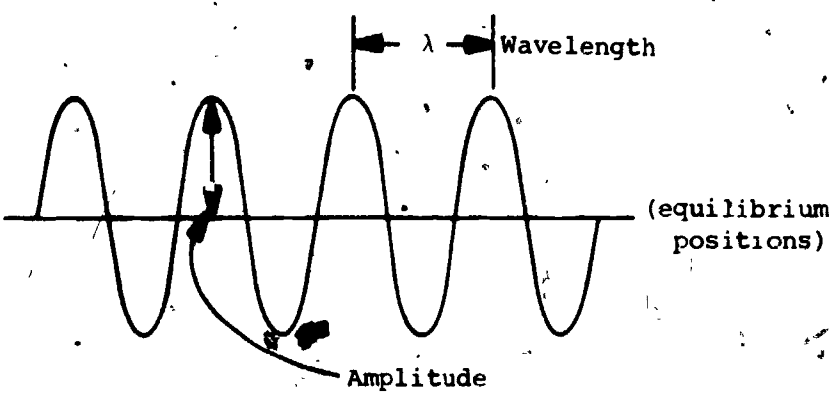
A transverse wave is a wave in which the motion of the particles of the medium is perpendicular to the motion of the wave (e.g. a wave in a string), producing a shape as shown below.



A longitudinal wave is a wave in which the motion of the particles of the medium is parallel to the motion of the wave (e.g. waves on a spring), producing a shape as shown below. (Sound waves are longitudinal waves.)



Wavelength is defined as the distance between successive crests - or troughs - (compressions - or rarefactions - in a longitudinal wave). See illustration below. It is usually symbolized λ (Greek lambda) and measured in meters in the metric system.



The wave speed v is the speed with which the crests and troughs (compressions and rarefactions in a longitudinal wave) move through the medium. It is determined by the physical properties of the medium.

The amplitude A of a wave is the maximum displacement of the particles of the medium from equilibrium position as the waves move through the medium. See illustration above.

The frequency, f, of a wave is the number of crests per second that pass a given point in the path of the wave. It is determined by the source of waves and measured in cycles per second (cps) or hertz (Hz). 1 cps = 1 Hz. (Dimensions of frequency are 1/s.)

The period T of a wave is the time between the passage of successive crests.

$$T = \frac{1}{4\ \text{Hz}} = 0.250\ \text{s}$$

The wave equation relates the variables of speed v, frequency f, and wavelength λ; for any wave:

$$v = \lambda f$$

For example, a 20 Hz wave with a 3 m wavelength must be moving with a speed

$$v = (3\ \text{m})(20\ \text{Hz}) = 60.0\ \text{m/s}$$

LABORATORY

The student should be able to directly measure the speed of a wave with appropriate length and time measuring devices. For example, he should be able to use a meter stick and stopwatch to measure the speed of a wave on a spring, or using appropriate equipment he should be able to measure the speed of sound in air. He should also be able to make direct measurements of wavelength and frequency when these are in the appropriate range.

WORKED EXAMPLES

1. A sound wave with frequency 440 Hz travels through the air and then into a swimming pool. What is its wavelength in each medium if the speed of sound in air is 340 m/s and in water is 1500 m/s?

   In air: $\lambda = \frac{v}{f} = \frac{340\ \text{m/s}}{440\ \text{Hz}} = 0.773\ \text{m}$

   In water only the speed changes (f determined by the source):

   $$\lambda = \frac{v}{f} = \frac{1500\ \text{m/s}}{440\ \text{Hz}} = 3.41\ \text{m}$$

2. If a water wave has a wavelength of 2 ft, and a speed of 5 ft/s, what is the period of the wave?

$$T = \frac{1}{f} \quad \text{and} \quad f = \frac{v}{\lambda}$$

therefore

$$T = \frac{\lambda}{v} = \frac{2\text{ ft}}{5\text{ ft/s}} = 0.400\text{ s}$$

STUDENT PROBLEMS

1. A sound wave with a 2 m wavelength in air (v = 340 m/s) travels into water (v = 1500 m/s). What is the period of the wave in each medium?

(5.88×10^{-3} s in both)

2. Which of the wave properties, amplitude, wavelength, speed, frequency, will directly affect the speed of the individual vibrating particles of the medium?

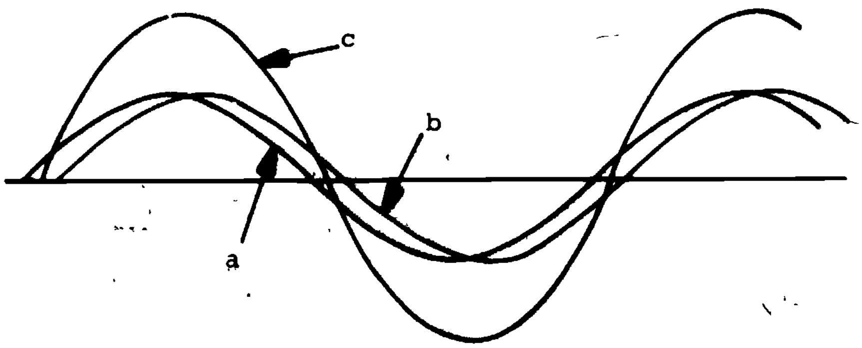
(amplitude & frequency)

3. Waves are put into a stretched spring by vibrating one end. They move down the spring with a speed of 6 ft/s and a wavelength of 1.5 ft. If the frequency is doubled by shaking the end twice as fast, what will be the new wavelength and speed?

(.750 ft; 6.00 ft/s)

SECTION 2 - SUPERPOSITION, STANDING WAVES, AND HARMONICS

The principle of superposition states that the displacement of a particle in the medium due to the presence of two waves at the same time is the sum of the displacements that each wave would produce singly. For example, the two waves a and b, shown below, will produce the wave form c.



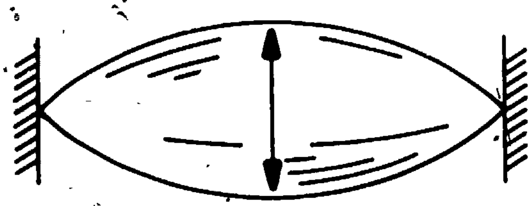
A standing wave on a string is a pattern of string vibration in which the string vibrates in the shape of a portion of a wave, with certain points never moving and other points undergoing maximum oscillation. Two traveling waves with the same amplitude and wavelength, traveling in opposite directions, will produce a standing wave.

A node is a point in a standing wave that has zero displacement at all times. A vibrating string fixed at both ends has nodes at its ends.

An antinode is a point in a standing wave that has maximum amplitude of oscillation.

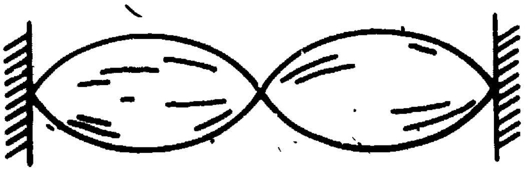
Modes of oscillation refer to the various shapes or waveforms with which a body can vibrate.

The fundamental mode of a string fixed at both ends is the simplest pattern of oscillation possible. It is shown below.

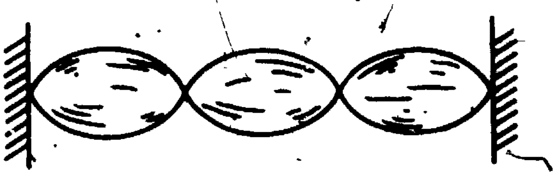


Fundamental

Harmonics of a string, fixed at both ends, are the basic waveforms with which the string can vibrate. The first harmonic is the fundamental and the others are often called overtones. The number of the harmonic gives the number of antinodes in the waveform. The second and third harmonics are shown below. A string may vibrate with more than one harmonic at a time, in which case its shape is determined by the Principle of Superposition.



Second Harmonic



Third Harmonic

The frequencies of harmonics are integral multiples of the fundamental frequency. If f_1 stands for the frequency of the fundamental, then the frequency, f_n, of the nth harmonic is

$$f_n = nf_1$$

For example, if the fundamental frequency is 220 Hz, the third harmonic frequency is

$$f_3 = 3 \times 220 \text{ Hz} = 660 \text{ Hz}$$

The string equation gives the fundamental frequency of a string fixed at both ends, whose mass m is in kg, length L in m, and which is under a tension T in N.

$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{m/L}}$$

For example, we may find the fundamental frequency of a string 1 m in length, whose mass is 10 g and is under a tension of 81 N.

$$f_1 = \frac{1}{2(1 \text{ m})}\sqrt{\frac{81 \text{ N}}{0.0100 \text{ kg}/ 1 \text{ m}}}$$

$$= (1/2)\sqrt{8100} \text{ Hz}$$

$$= (1/2)(90 \text{ Hz}) = 45.0 \text{ Hz}$$

LABORATORY

The student should be able to verify the string equation for something like a guitar string using suspended weights, meter stick, and a frequency determining device such as a stroboscope or an oscilloscope.

WORKED EXAMPLES

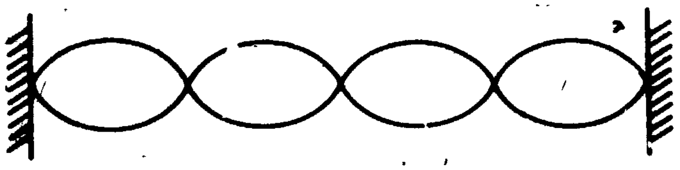
1. Find the frequency of the fourth harmonic for an 80 g, 2 m long string fixed at both ends and under a tension of 144 N.

$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{m/L}} = \frac{1}{4 \text{ m}}\sqrt{\frac{144 \text{ N}}{80.0 \times 10^{-3} \text{ kg}/2 \text{ m}}}$$

$$= 15.0 \text{ Hz}$$

$$f_4 = 4f_1 = (4)(15.0\ \text{Hz}) = 60.0\ \text{Hz}$$

2. If the string below has a fundamental frequency of 330 Hz, with what frequency is it vibrating as shown?

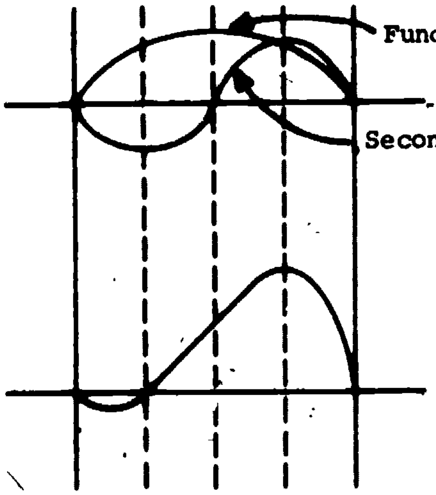


4 antinodes n = 4 (fourth harmonic)

$$f_4 = (4)(330\ \text{Hz}) = 1320\ \text{Hz} = 1.32 \times 10^3\ \text{Hz}$$

3. Show the shape of the waveform on a string if it is vibrating with its fundamental and second harmonic simultaneously. The amplitude of the second harmonic is half that of the amplitude of the fundamental.

Here we must sketch the two waves and roughly add them using the Principle of Superposition.



STUDENT PROBLEMS

1. Doubling the tension on a string fixed at both ends, has what effect on its frequency?

   (Increased by a factor 1.41)

2. A string 40 cm long and with a mass of 36 g is put under a tension of 490 N. What is its fundamental frequency? What is the fundamental frequency of a sample of the same string twice as long and under the same tension?

   (92.2 Hz; 46.1 Hz)

3. If a string, fixed at both ends, is vibrating with a node at its center, what harmonics cannot be present?

   (All odd harmonics)

4. If the fundamental frequency of a string is 200 Hz, sketch the harmonic whose frequency is 600 Hz.

5. Sketch the shape of a string vibrating with its fundamental and its third harmonic simultaneously. The amplitude of the third harmonic is 1/2 the amplitude of the fundamental.

SECTION 3 - SOUND POWER, INTENSITY, AND DECIBELS

The power of a wave, or the energy carried per unit time, is proportional to the square of the wave amplitude A.

$$P \propto A^2$$

For example, doubling the amplitude of a wave increases its power by a factor of four.

The intensity of a wave is defined as the power it carries per unit area. If P represents the power that a wave carries through an area a perpendicular to the wave motion, then intensity I is given by

$$I = P/a$$

For example, a sound wave that carries a power of 5×10^{-3} watts through a window of area 2 m^2 has an intensity of

$$I = \frac{5 \times 10^{-3}\ W}{2\ m^2} = 2.50 \times 10^{-3}\ W/m^2$$

The inverse square law states that the intensity of a wave in space decreases as $1/d^2$ where d is the distance from the source.

$$I \propto 1/d^2$$

Thus, doubling the distance from the source decreases the wave intensity by a factor of four.

The intensity level (I.L.) is proportional to the common logarithm of the intensity and is a measure of loudness at a single frequency. In decibels, the intensity level is given by

$$I.L. = 10 \log (I/I_o)$$

where I is the intensity of the sound wave whose intensity level is being calculated, and $I_o = 10^{-12}$ W/m^2 is the intensity of the minimum audible sound.

For example, the intensity level of the sound wave in the last example, is

$$I.L. = 10 \log (2.5 \times 10^{-3}/10^{-12})$$

$$= 10 \log (2.5 \times 10^9)$$

$$= 10 (\log 2.5 + \log 10^9) = 10 (.398 + 9)$$

$$\cong 94 \text{ dB}$$

In approximate terms, a change of about +3 dB corresponds to a doubling of the intensity I; -3 db, of course, implies a halving of the intensity I. If the intensity level is raised by another +3 db, the intensity again doubles - it becomes four times what it was originally; that is, +6 dB implies an intensity four times as great; and so on. Each successive doubling means a change of +3db..

LABORATORY

The student should be able to use a sound level meter to map out a pattern of intensity level from a speaker or other source. He should also be able to verify the inverse square law.

WORKED EXAMPLES

1. If the intensity level at one position is 75 dB, what is the intensity level at a point twice as far from the source of sound?

   If we use subscripts 1 and 2 to represent the first and second positions respectively, we can write

$$IL_1 - IL_2 = 10 \log(I_1/I_o) - 10 \log (I_2/I_o)$$

$$= 10 \left[\log(I_1/I_o) - \log(I_2/I_o)\right]$$

$$= 10 \log\left[(I_1/I_o)/(I_2/I_o)\right]$$

$$IL_1 - IL_2 = 10 \log (I_1/I_2)$$

But we know $IL_1 = 75$ dB and the inverse square law says $(I_1/I_2) = 4$.

$$IL_2 = 75 \text{ db} - (10 \log 4) \text{ db} = 75 \text{ db} - 6 \text{ db} = 69 \text{ dB}$$

or using the approximation above, for a case of cutting the intensity by a factor of 4 (= 2 x 2), the intensity level goes down by 6 db (= 2 x 3 db).

2. One sound wave produces a sound level meter reading of 60 dB. What would be the reading if the amplitude of the wave were multiplied by four?

We may use the formula from the previous example.

$$IL_1 - IL_2 = 10 \log(I_1/I_2)$$

Here $IL_1 = 60$ dB and $I_1/I_2 = 1/16$ because $I \propto P \propto A^2$.

$$IL_2 = 60 - 10 \log(1/16)$$

$$= 60 + 10 \log(16) = 60 + 12$$

$$= 72 \text{ dB}$$

or, using the approximation above, for a case of multiplying the amplitude by four (intensity up by a factor of $4^2 = 16$), the intensity level goes up by (4)(3 db) = 12 db.

STUDENT PROBLEMS

1. If the "window" to your ear is 1 cm^2 in area, what power is collected by your ear from the minimum audible sound wave with intensity I_o?

   (10^{-16} W)

2. What is the intensity level of a sound with 1000 times the intensity of the minimum audible sound?

   (30 dB)

3. A speaker produces a sound level meter reading of 82 dB at a distance of 5 m. What would be the reading if the sound power output of the speaker is doubled?

   (85 dB)

4. A sound level meter reads 80 dB at a point 6 m from a speaker producing a constant tone. How far away must the meter be moved to give a 70 dB reading, assuming no echoes and no other sources?

(19.0 m)

5. A speaker produces a sound level meter reading of 50 dB a certain distance away. What will be the reading three times as far from the source if the speaker is turned up to put out a wave with five times the amplitude?

(54.4 dB)

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